Creating Reasoning Routines, Building Problem-Solvers

Session 1

Whole Class Routines

Deconstructing Word Questions

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Y5 & Y6

Content Knowledge

Recall times tables

Place value:

32 = 30 + 2

32 = 20 + 12

Convert the denominators to add fractions

1kg = 1000g

Area = length × width

Learning Dispositions

Derive facts

See connections between real life and mathematics

Follow other perspectives

Persevere through challenge

Explain/demonstrate understanding

Create own examples

Routines Within Interactive Teaching

Raising the internal narrative:

- Gap between question and response/discussion
- Silence in modelling

I already…

I will try…

A context for…

A sceptic would say…

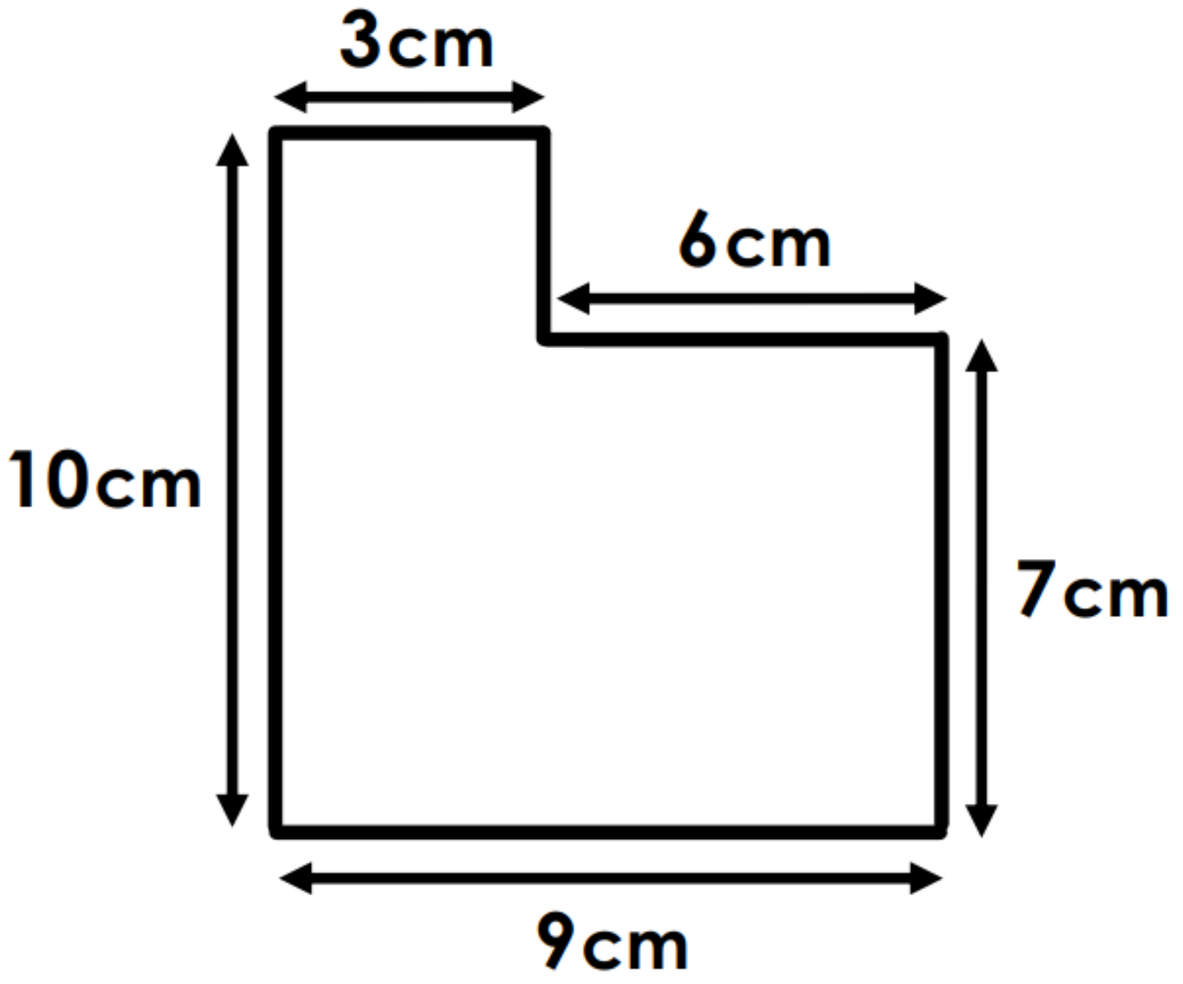
$$\begin{array}{r} 63 \\ \times\ 53 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ \times\ 53 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 63 \\ \times\ 53 \\ \hline 189 \end{array}$$

$$
\begin{array}{r}
63 \\
\times\ 53 \\
\hline
189 \\
15
\end{array}
$$

Calculate the area of the shape:



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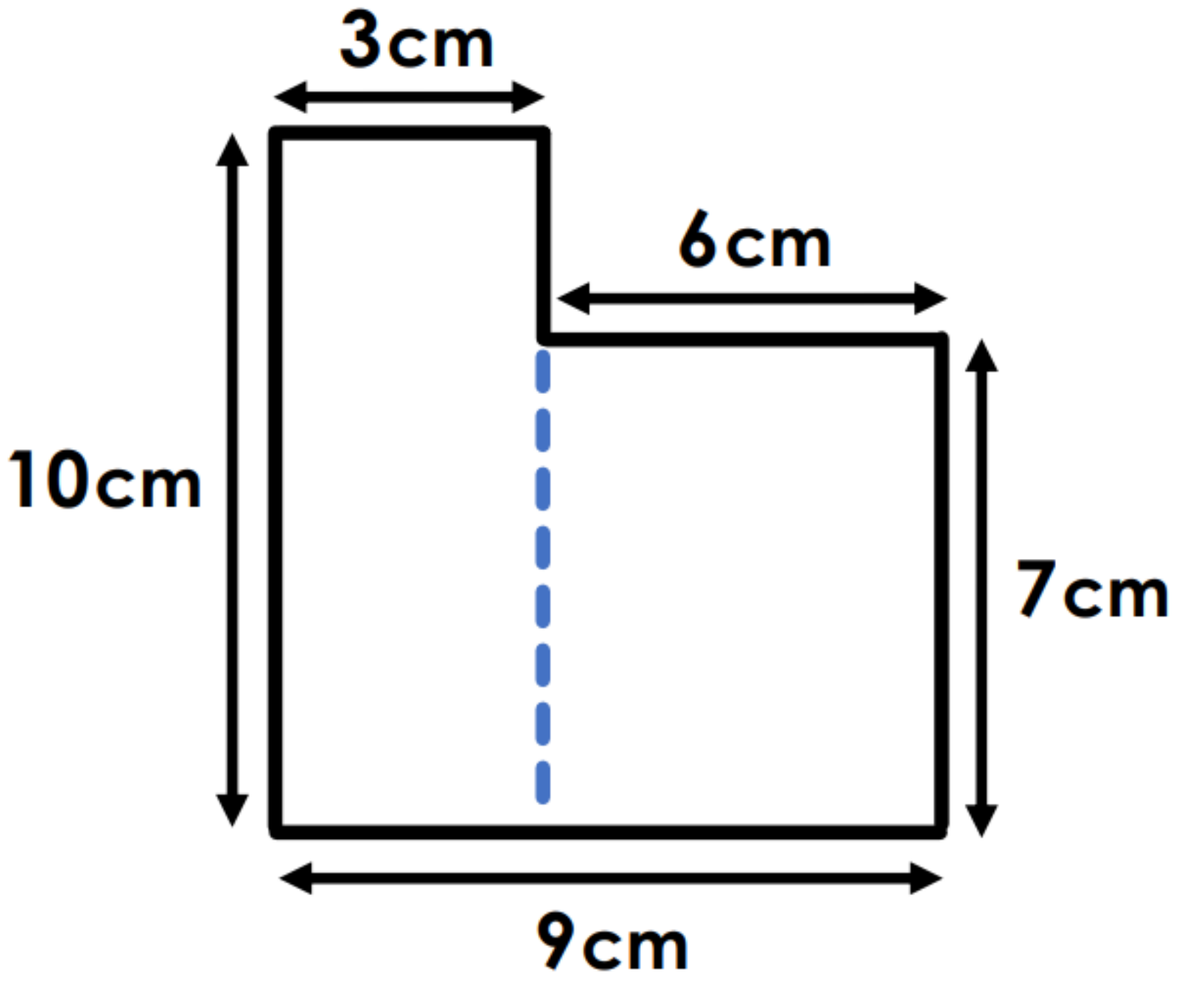

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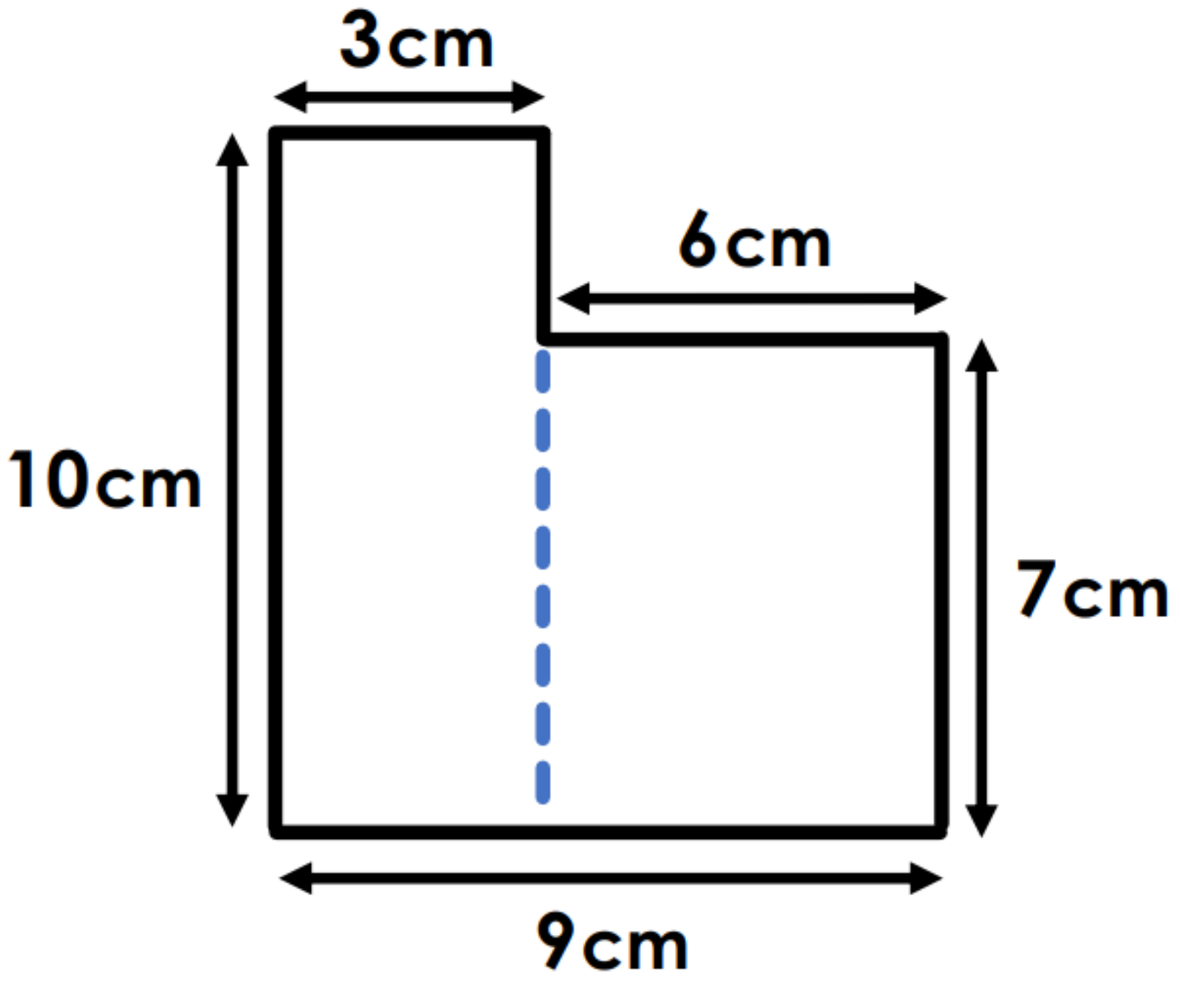
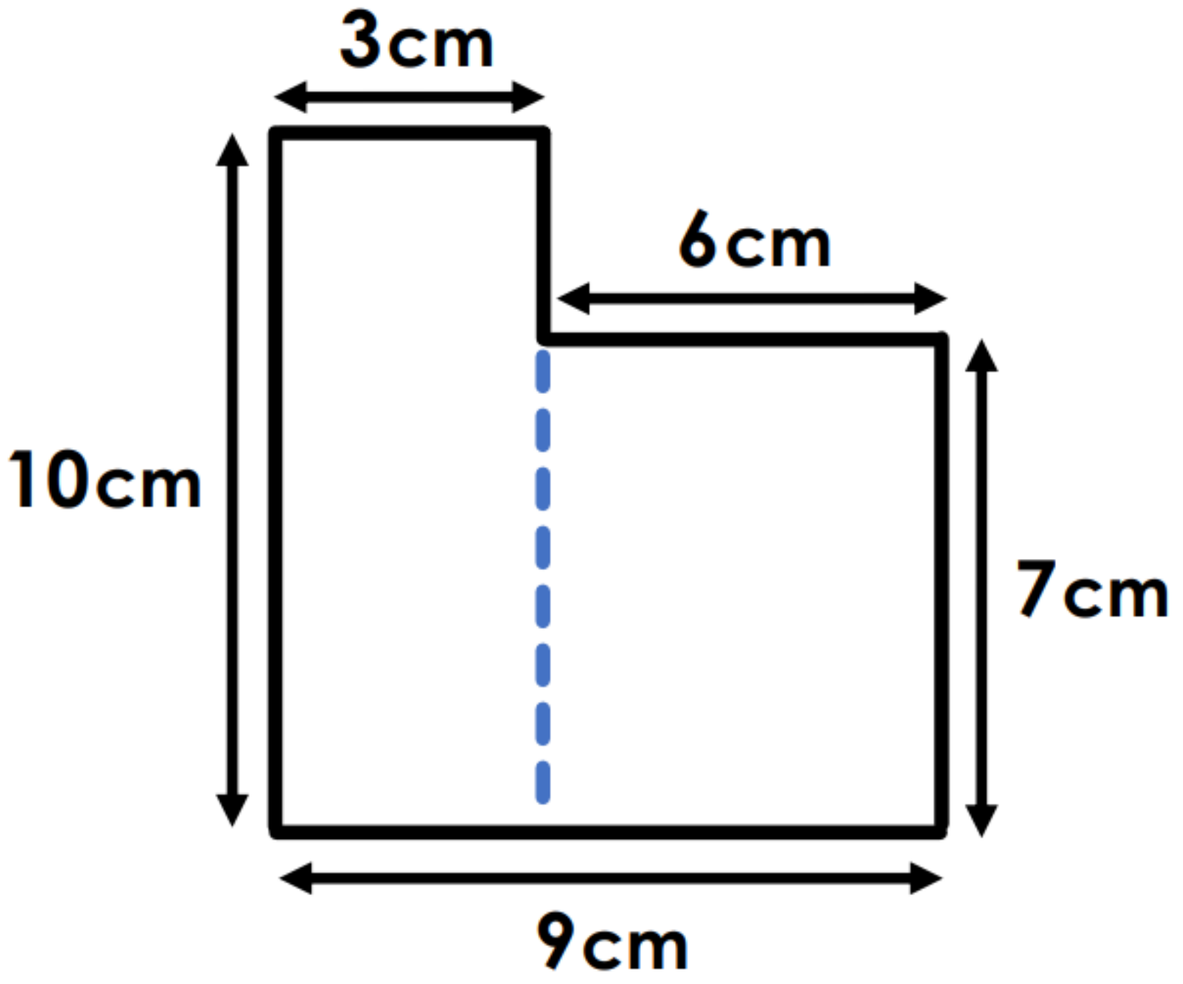


$3\text{cm} \times 10\text{cm} = 30\text{cm}^2$

Calculate the area of the shape:



$3\,cm \times 10\,cm = 30\,cm^2$

$7\,cm \times 9\,cm = 63\,cm^2$

Which Answer?

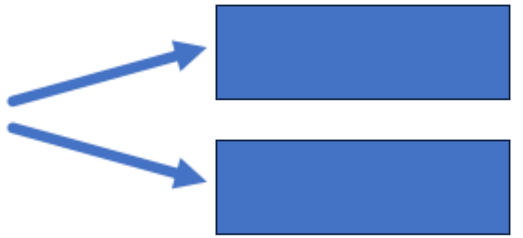
Write the number nine thousand and six

Max: 90006 **Raja:** 9006

Explain the mistake

Which Answer?

Three thousand and four



50070

Which Answer?

Three thousand and four

- **30 004**
- **3 004**

50070

Which Answer?

Three thousand and four → **30 004**

Three thousand and four → **3 004**

50070 → **Fifty thousand and seventy**

50070 → **Five hundred and seventy**

Zara's book is **60** pages long.

Zara has read $\frac{3}{4}$ of her book.

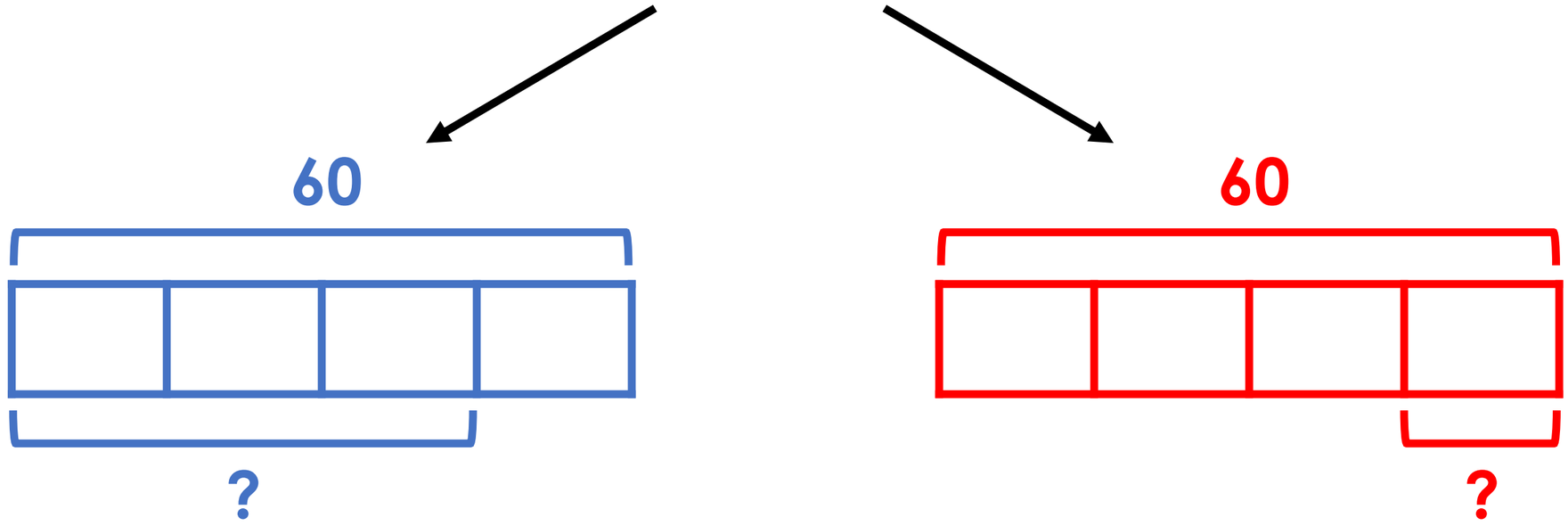
How many pages does Zara have left to read?

Zara's book is **60** pages long.

Zara has read $\frac{3}{4}$ of her book.

How many pages does Zara have left to read?

Which bar model represents the question?



Routines Within Interactive Teaching

Raising the internal narrative:

- Gap between question and response/discussion
- Silence in modelling

Mass participation:

- Form of answer before question
- Explicit and shared routines
- Depth

I already…

I will try…

A context for…

A sceptic would say…

For each example, **can the mean be calculated?**
Why would the mean be calculated?

The mean number of days in a week.

The mean number of baskets scored by a netball player per match.

The mean price of a mango in the supermarket.

The mean happiness you feel each morning.

The mean height of an adult giraffe.

The Mean

Task A

Tick the examples where **the mean can be calculated and is useful.**

(a) What is the mean weight of a new-born baby? ✓

(b) A pack of 6 eggs costs £1.20. What is the mean cost of each egg? ~~✓ but not very~~ ×

(c) How friendly is the average dog? Give the answer as a mean. ×

(d) What is the mean amount of electricity used by each house per month? ✓

For one example, explain how calculating the mean could be useful:

(A) The mean of a new born baby is useful to know because then you can find out if a baby is heavy or light.

Which Answer?

Here are the shoe sizes for five children:

3, 4, 3, 7, 3

The mean shoe size is size 3

The mean shoe size is size 4

Which Answer?

Here are the shoe sizes for five children:

3, 4, 3, 7, 3

The mean shoe size is size 3

The mean shoe size is size 4

Shoe sizes	**3**	**4**	**3**	**7**	**3**

Which Answer?

Here are the shoe sizes for five children:

3, 4, 3, 7, 3

✗

The mean shoe size is size 3

✓

The mean shoe size is size 4

Shoe sizes	**3**	**4**	**3**	**7**	**3**
Mean	**4**	**4**	**4**	**4**	**4**

Which Answer?

Three numbers have a mean of 12.
What could the numbers be?

9, 11 and 16

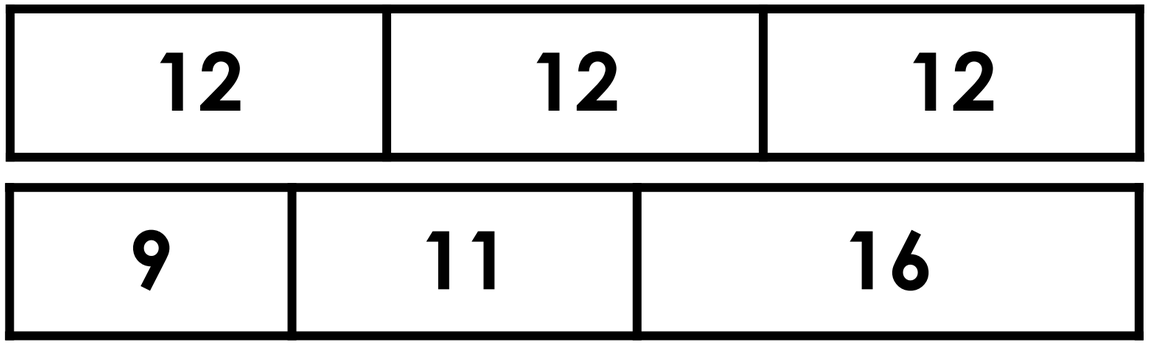
6, 2 and 4

Which Answer?

Three numbers have a mean of 12.
What could the numbers be?

9, 11 and 16

6, 2 and 4



Which Answer?

Three numbers have a mean of 12.
What could the numbers be?

9, 11 and 16 ✓

6, 2 and 4 ✗

12	12	12
9	11	16

The Mean

Task B

Explain the mistakes. Give the correct answer.

What is the **mean** of **7, 5, 9** and **3?** **7 + 5 + 9 + 3 = 24** **24 ÷ 3 = 8**	*The mistake is...* They divided by 3 not 4! *Answer:* 6
Give **3 numbers** with a mean of **20.** **8, 5 and 7**	*The mistake is...* If you add 8,5 and 7 you'd get 20 but you need to divide. *Answer:* 17, 23, 20
Adult cinema tickets: **£9** each Child cinema tickets: **£5** each **3 adults** and **1 child** go to the cinema. **What is the mean cost of each ticket?** **£9 + £5 = £14** **£14 ÷ 2 = £7**	*The mistake is...* They added 9 and 5 instead of 9, 9, 9 and 5. *Answer:* £8

Example 1:

30, 5, 1

Example 2:

12, 13, 11

Interesting Example:

18.7, 0.3, 17

Non-Example

4, 3, 5

Rank by Difficulty

20% of 440

40% of 220

15% of 300

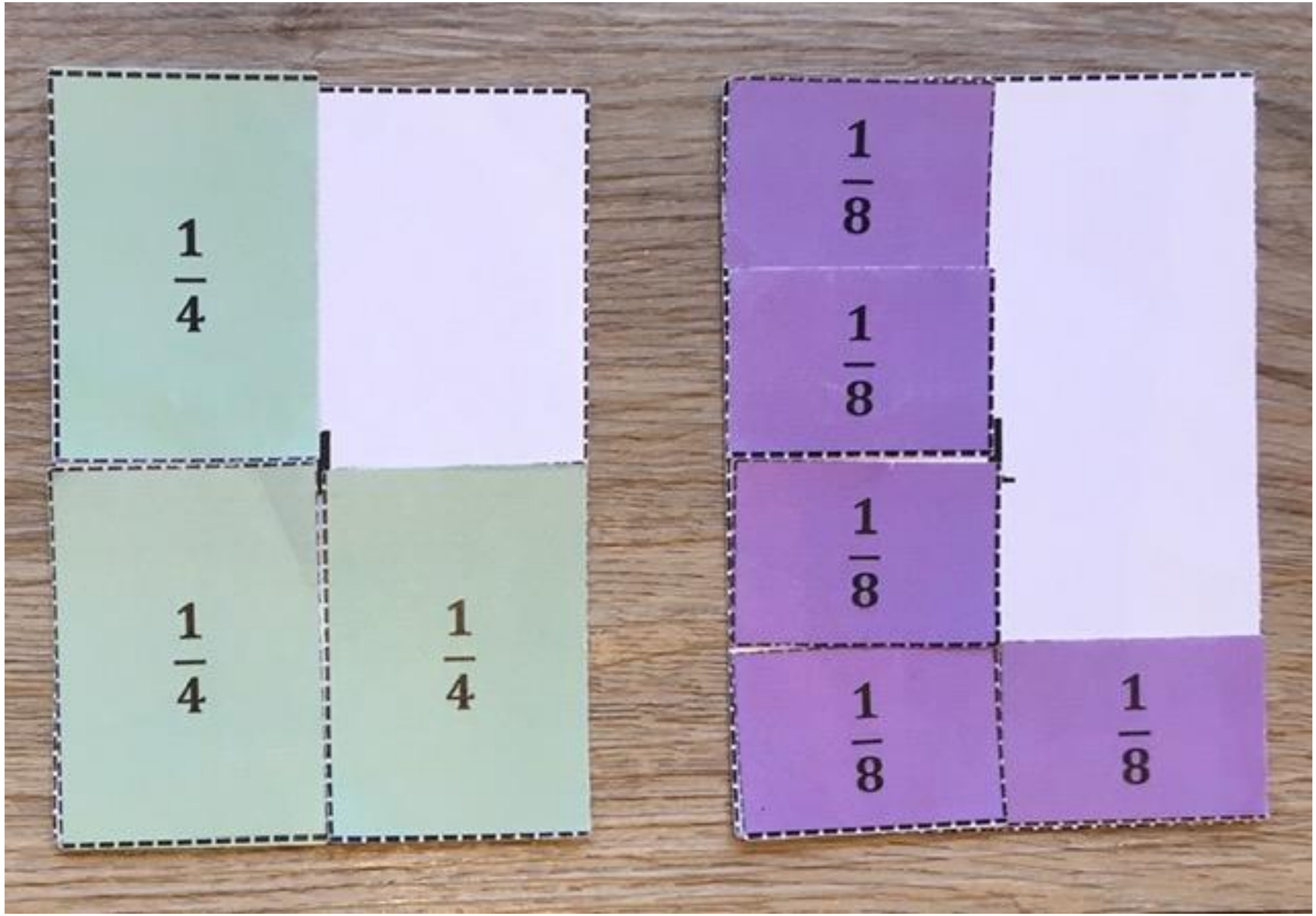
Rank by Difficulty

$$\frac{3}{7} + \frac{3}{7}$$

$$\frac{4}{10} + \frac{1}{5}$$

$$\frac{4}{8} + \frac{3}{6}$$

$$\frac{3}{4} + \frac{5}{8}$$



Explain the Mistakes

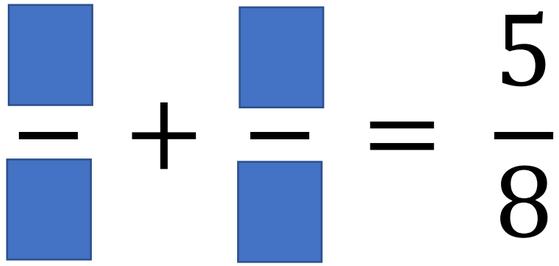
Mistake A:

$$\frac{1}{4} + \frac{3}{8} = \frac{4}{12}$$

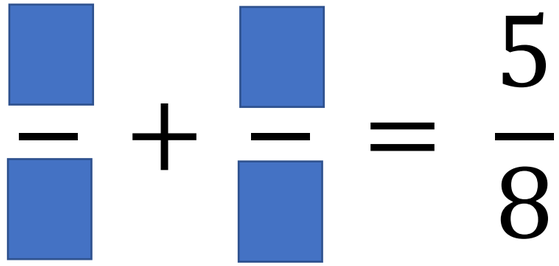
Mistake B:

$$\frac{1}{4} + \frac{3}{8} = \frac{4}{8}$$

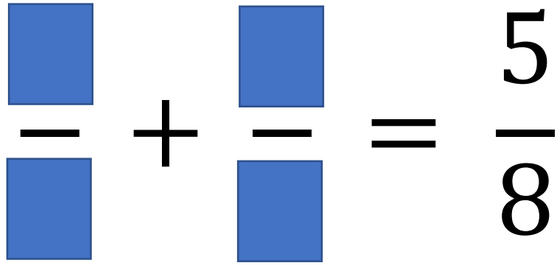
Example 1



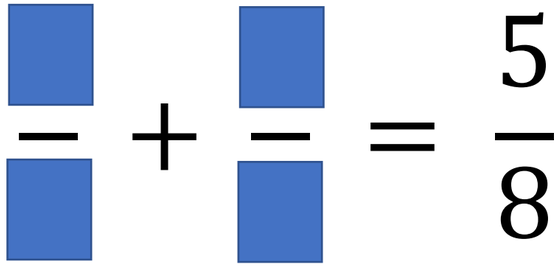
Example 2



Interesting Example



Non-example



1

$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ ✓

2

$\frac{4}{16} + \frac{6}{16} = \frac{5}{8}$

$(\frac{2}{8}) + (\frac{3}{8})$ ✓

Interesting

$\frac{1}{2} + \frac{2}{16} = \frac{5}{8}$

$(\frac{4}{8}) + (\frac{1}{8})$ ✓

Non

$\frac{1}{4} + \frac{4}{4} = \frac{5}{8}$ ✗

Routines Within Interactive Teaching

Raising the internal narrative:

- Gap between question and response/discussion
- Silence in modelling

Mass participation:

- Form of answer before question
- Explicit and shared routines
- Depth

Managing discussions:

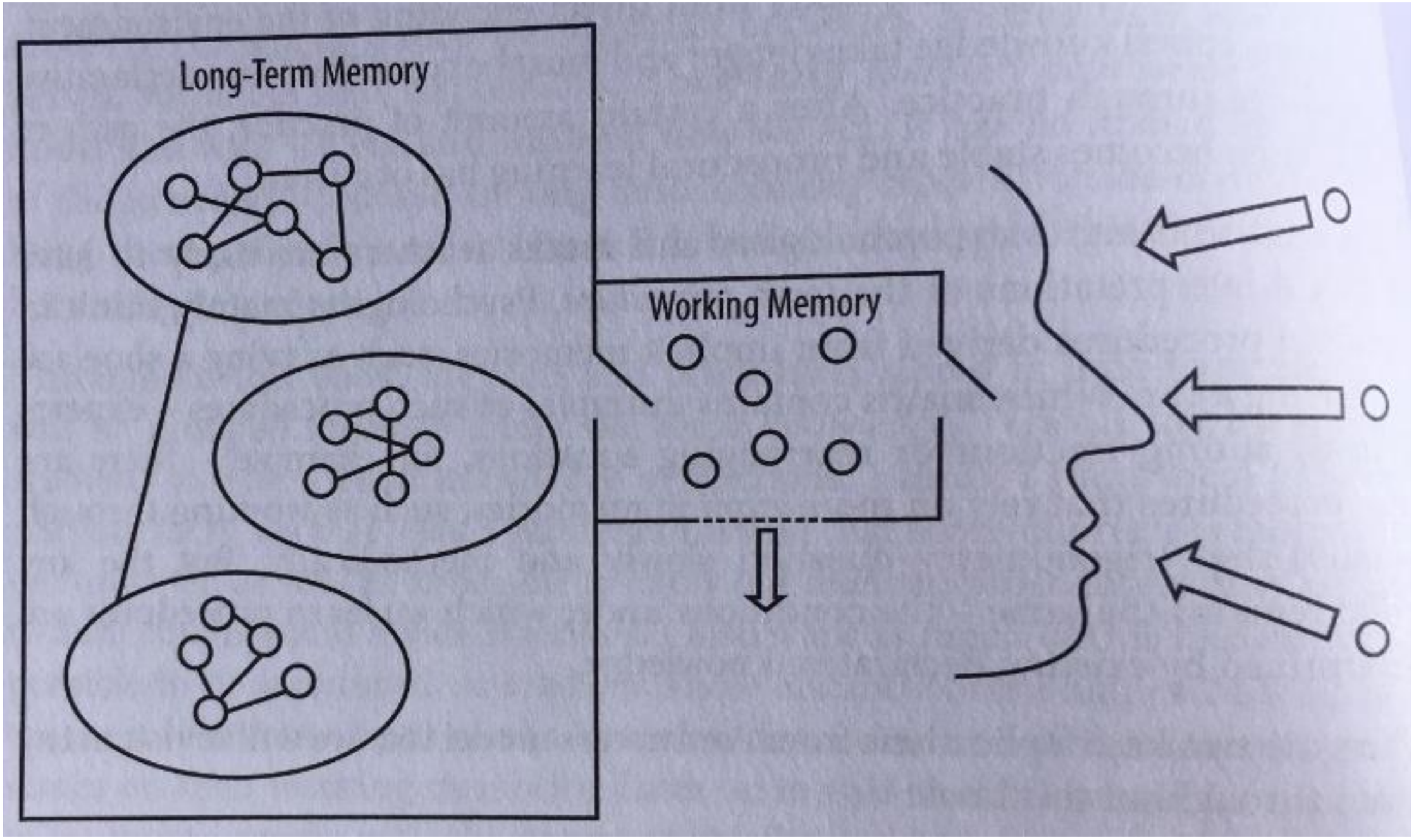
- Selecting and priming responses
- Wait time 2, following perspectives
- ‘Say it again, better’

I already…

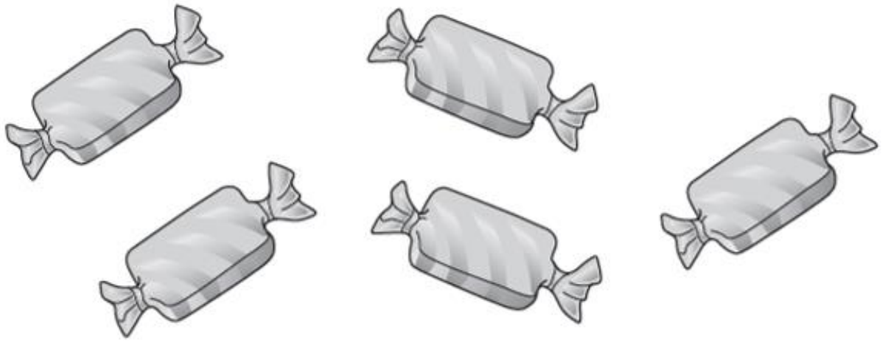
I will try…

A context for…

A sceptic would say…


Long-Term Memory
Working Memory

Bilal spends **25p** on these sweets:



Each sweet costs the same amount.

Work out the cost of **3** of these sweets.

There are 432 places at a dance school.

There are two age groups.

This table shows the number of classes and the number of pupils in each class for each age group at the moment.

Age in years	Number of classes	Number of pupils in each class
7–12	15	16
13–18	10	18

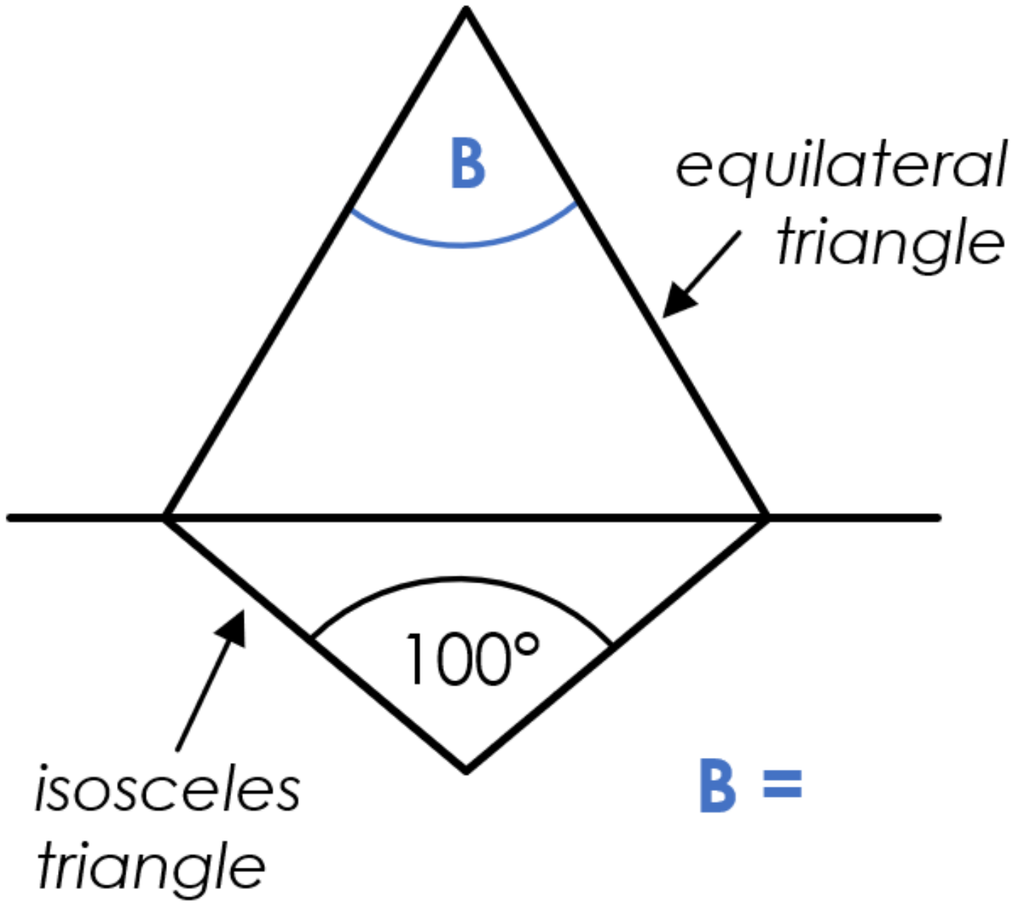
How many **more** pupils can join the dance school?

Jen has 6 stickers.

Helen has 4 stickers.

In total, they have 10 stickers.

How many more stickers does Jen have than Helen?



When students are presented with a mathematics word problem, their first response often is to try to compute an answer, even before they have tried to understand the problem.

Expert problem solvers typically spend more time thinking about problems and trying to understand them than do novices, who tend to immediately execute a solution.

Removing Opportunities to Calculate Improves Students' Performance on Subsequent Word Problems. *Givvin and Stigler (2019)*

A group of tourists planned a 3-day walking trip from Big Rock to Eagles Landing, a total of 66 km. On the first day they walked 22 km. On the second day they walked 20 km.
How far would they have to walk on the third day of their trip?

A group of tourists planned a 3-day walking trip from Big Rock to Eagles Landing. On the first day they walked one third of the total distance. On the second day they walked a little less.
How far would they have to walk on the third day of their trip?

Removing Opportunities to Calculate Improves Students' Performance on Subsequent Word Problems. *Givvin and Stigler (2019)*

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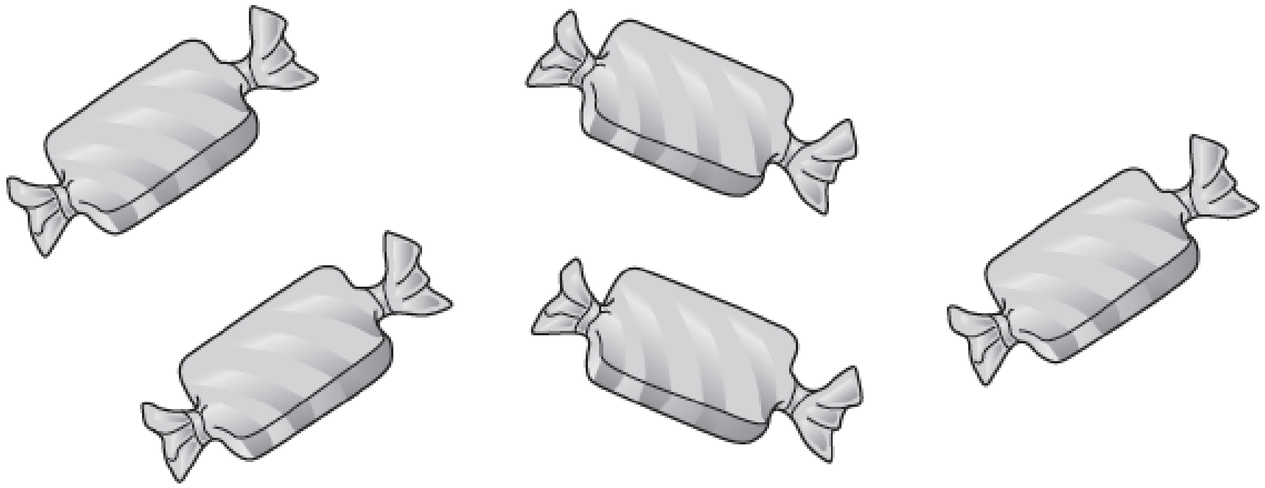

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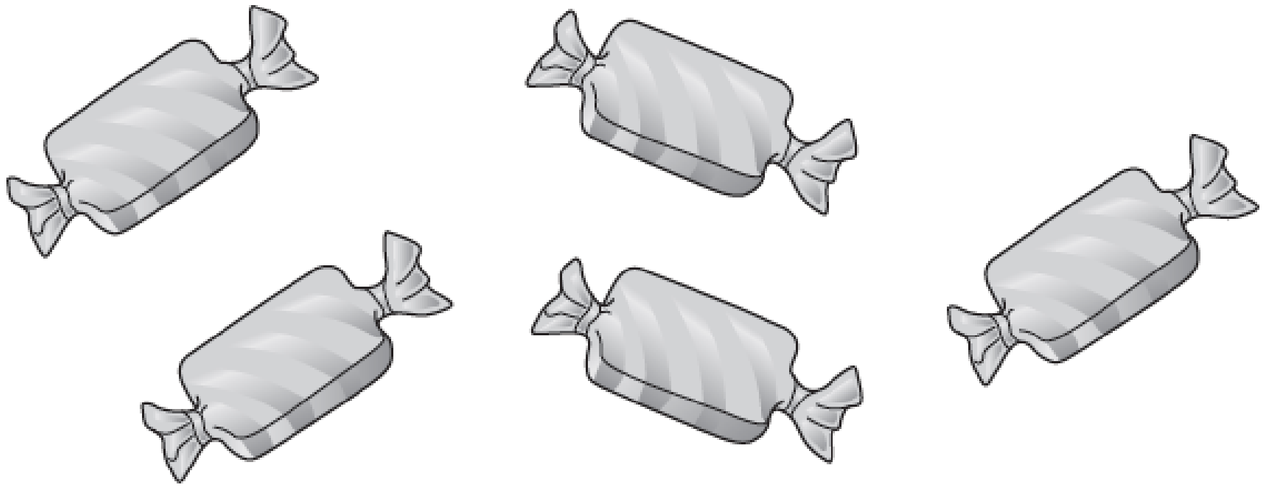

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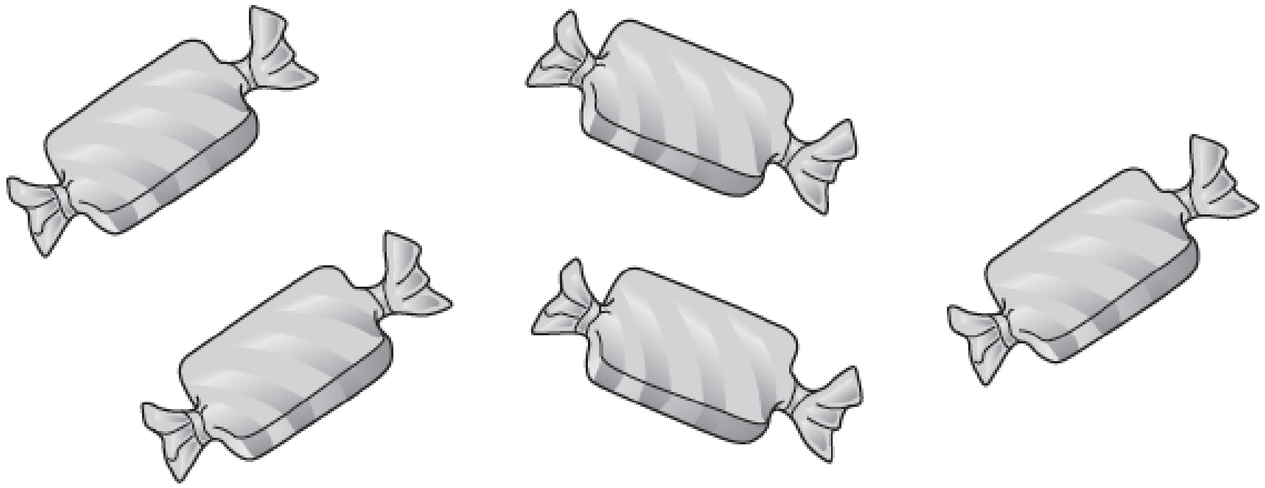

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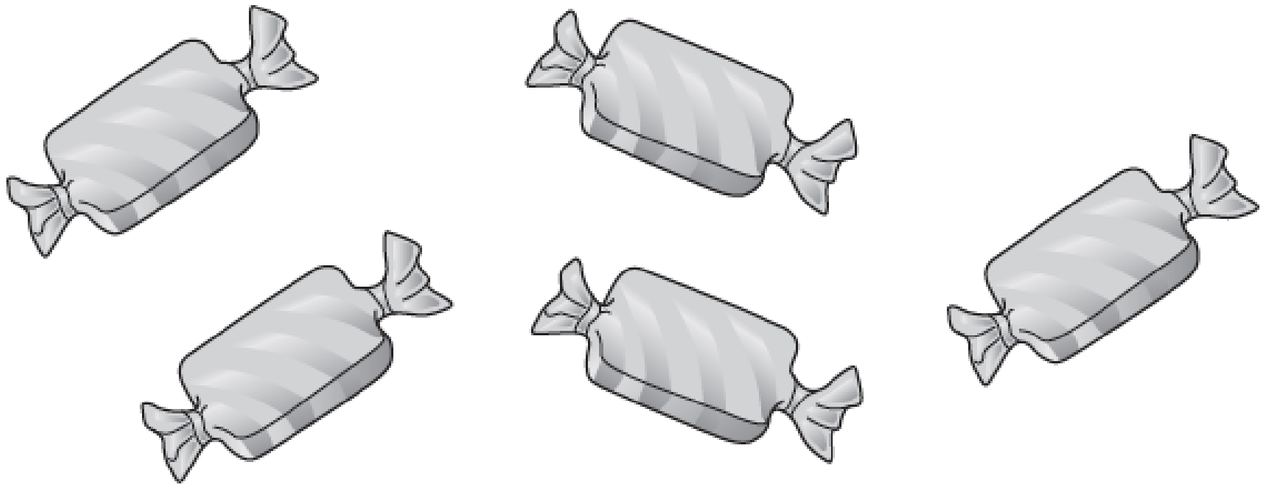

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How many **more** pupils can join the dance school?

[redacted] at a dance school.

There are two age groups.

Age in years	Number of classes	Number of pupils in each class
7–12	8	12
13–18	10	18

There are 432 places at a dance school.

There are two age groups.

Age in years	Number of classes	Number of pupils in each class
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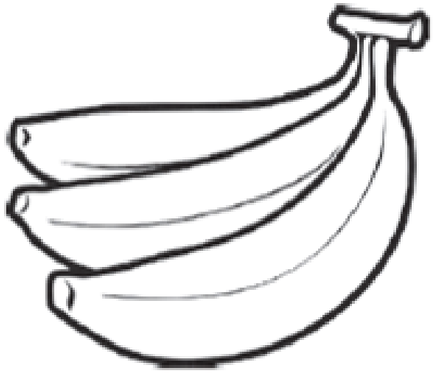
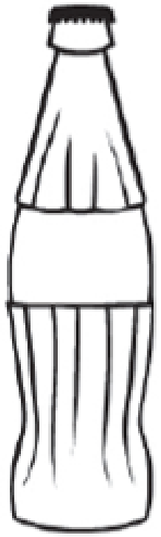
Age in years	Number of classes	Number of pupils in each class
7–12	8	12
13–18	10	18

How many **more** pupils can join the dance school?

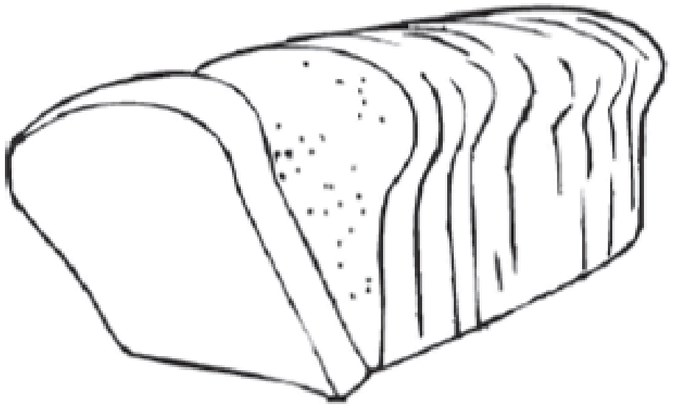
Here is the cost of some items in a shop.



Banana 20p

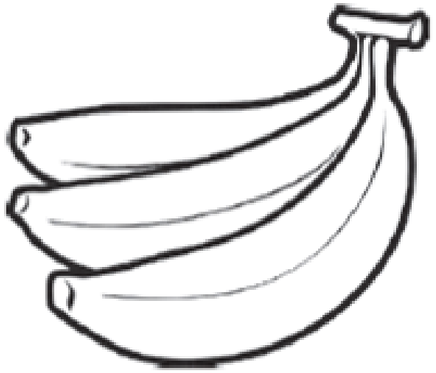


Drink 50p

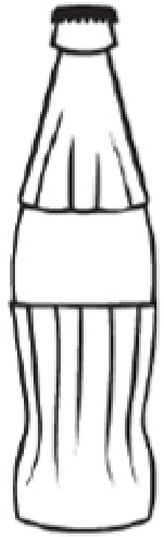


Bread 40p

Here is the cost of some items in a shop.



Banana 20p



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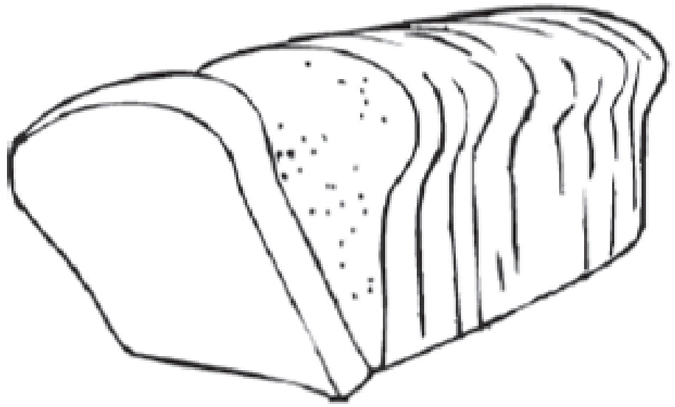
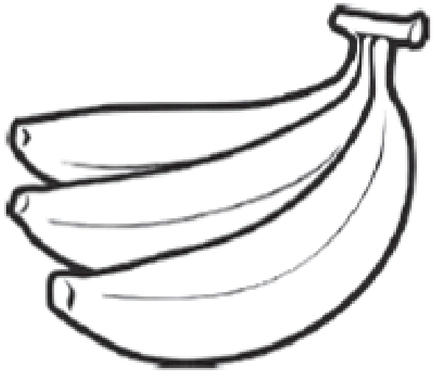


Bread 40p

How many bananas can he buy?

Here is the cost of some items in a shop.



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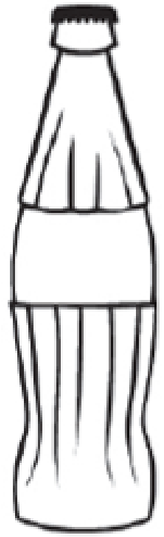

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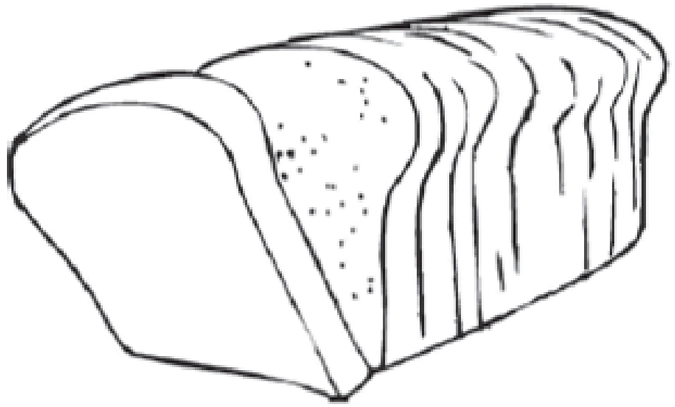
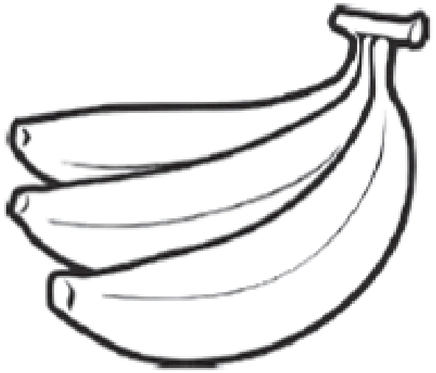


Bread 40p

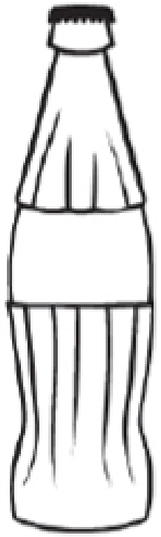
Sam has £1.

How many bananas can he buy?

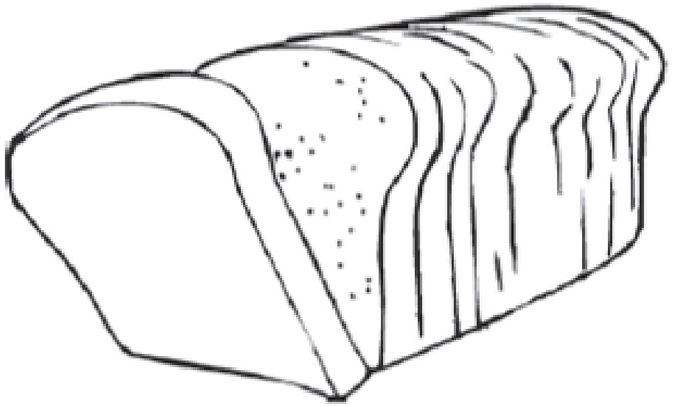
Here is the cost of some items in a shop.



Banana 20p



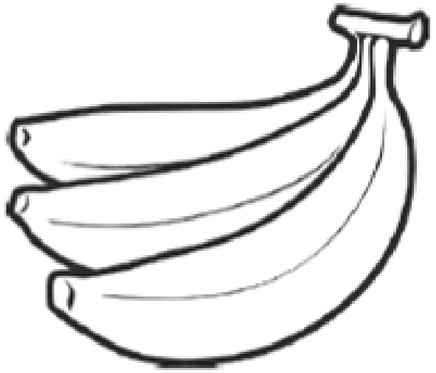
Drink 50p



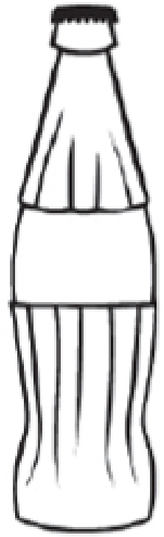
Bread 40p

How much change does she get?

Here is the cost of some items in a shop.



Banana 20p



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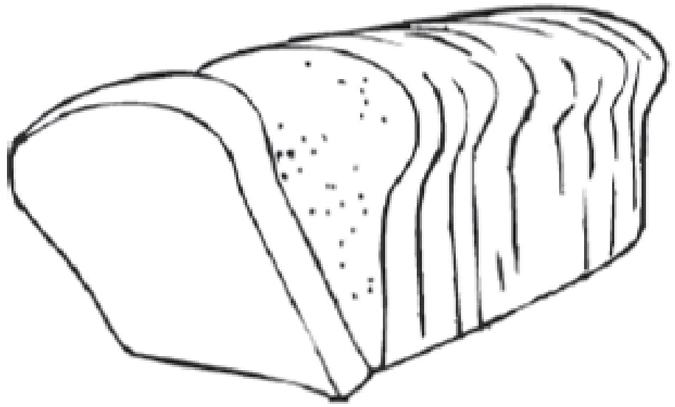
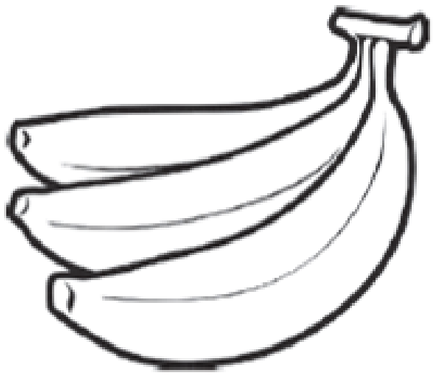


Bread 40p

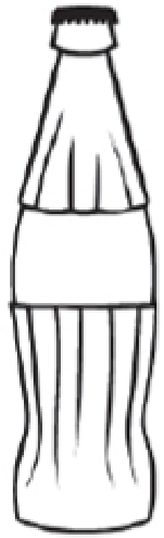
Jen buys a loaf of bread and two drinks.

How much change does she get?

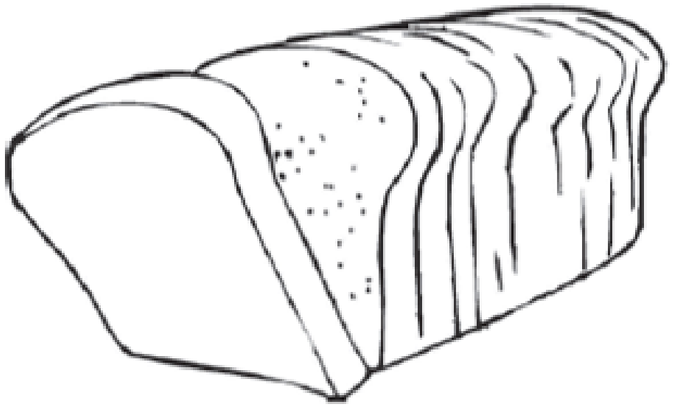
Here is the cost of some items in a shop.



Banana 20p



Drink 50p



Bread 40p

Jen buys a loaf of bread and two drinks.

She pays with a £2 coin.

How much change does she get?

A circus is holding a concert for charity.

Adult tickets cost **£11**. Child tickets cost **£6**.

How many child tickets are sold?

What information must be given?

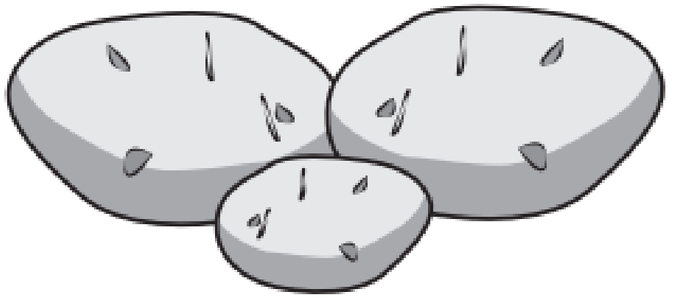
A circus is holding a concert for charity.

Adult tickets cost **£11**. Child tickets cost **£6**.

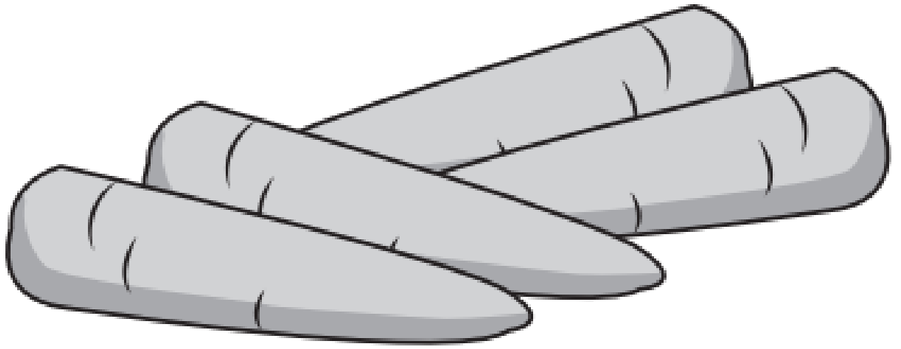
120 adult tickets are sold. In total, **£1800** is raised.

How many child tickets are sold?

16



potatoes
£1.50 per kg



carrots
£1.80 per kg

Jack buys $1\frac{1}{2}$ kg of potatoes and $\frac{1}{2}$ kg of carrots.

How much **change** does he get from **£5**?

Sports Direct

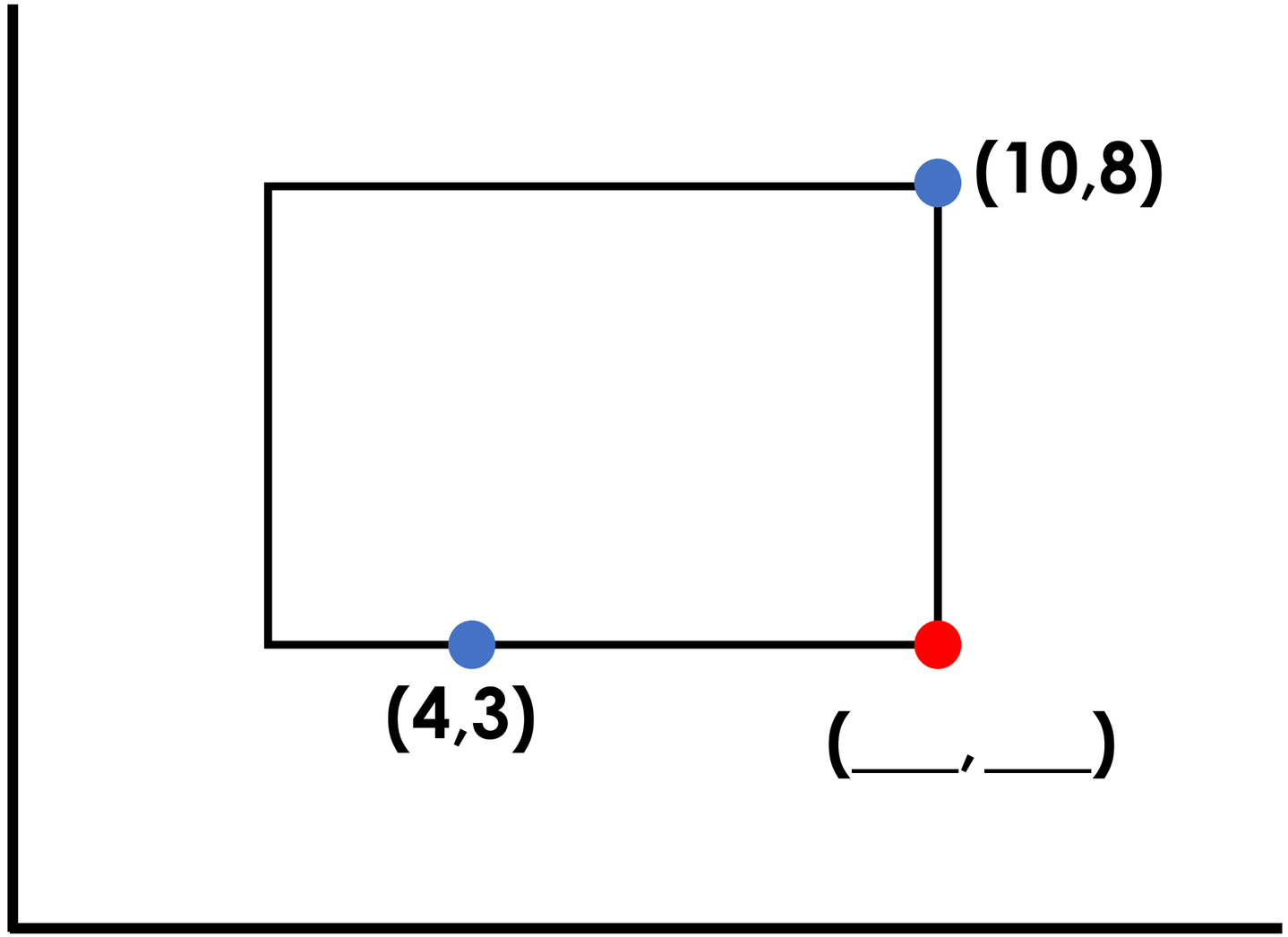
Tennis balls: £1.50 each

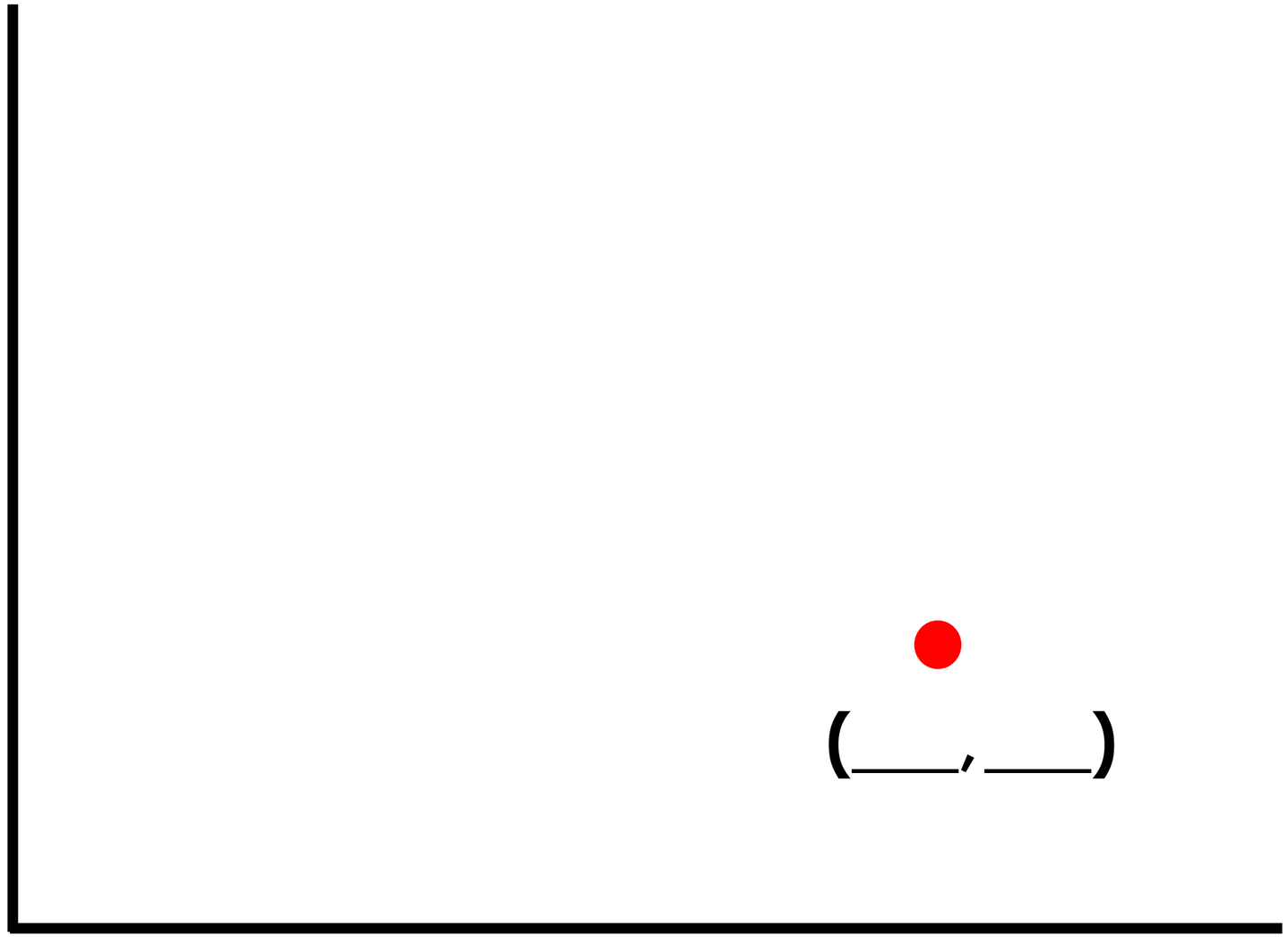
JD Sports

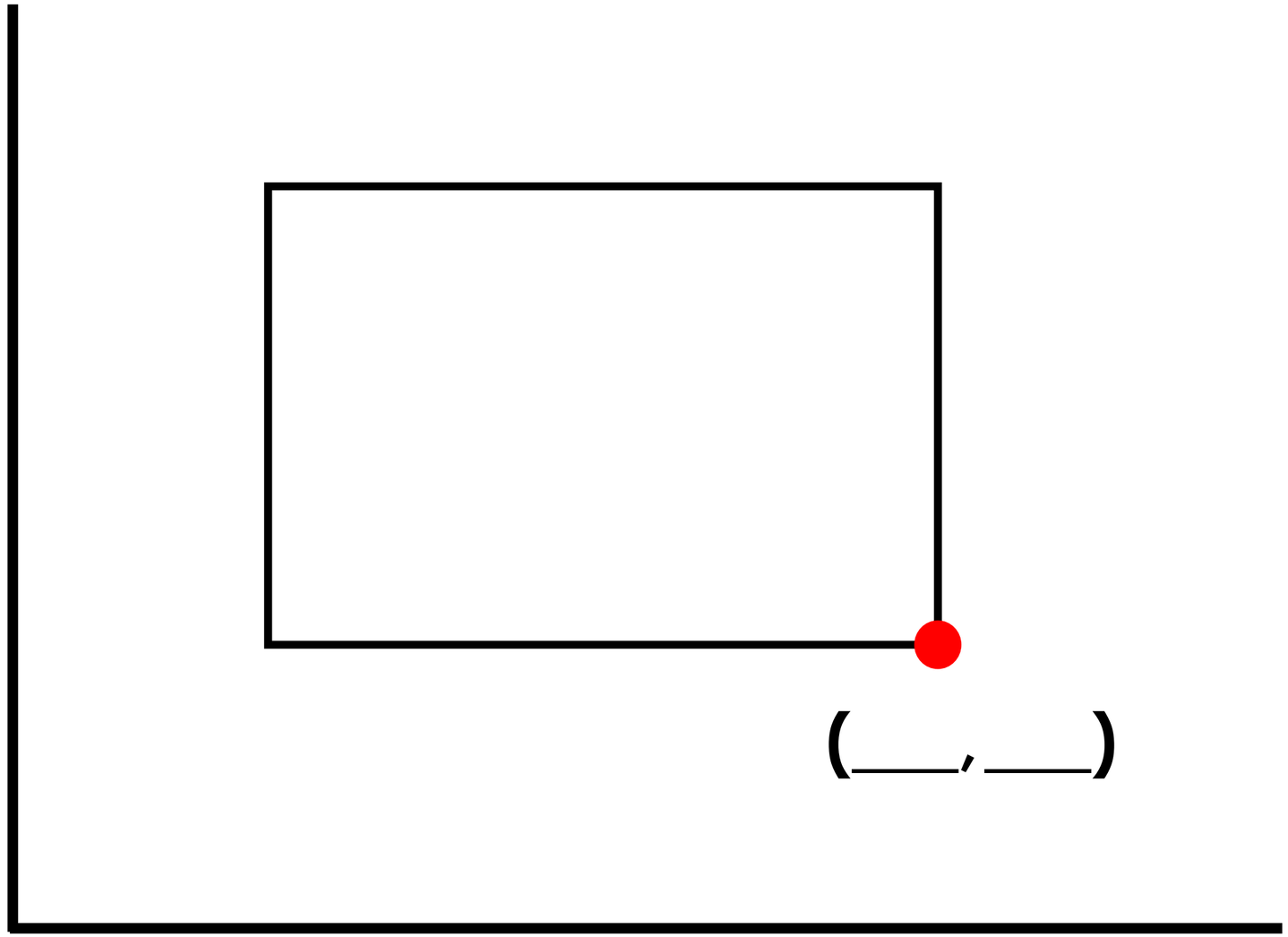
Tennis balls: £5 for four

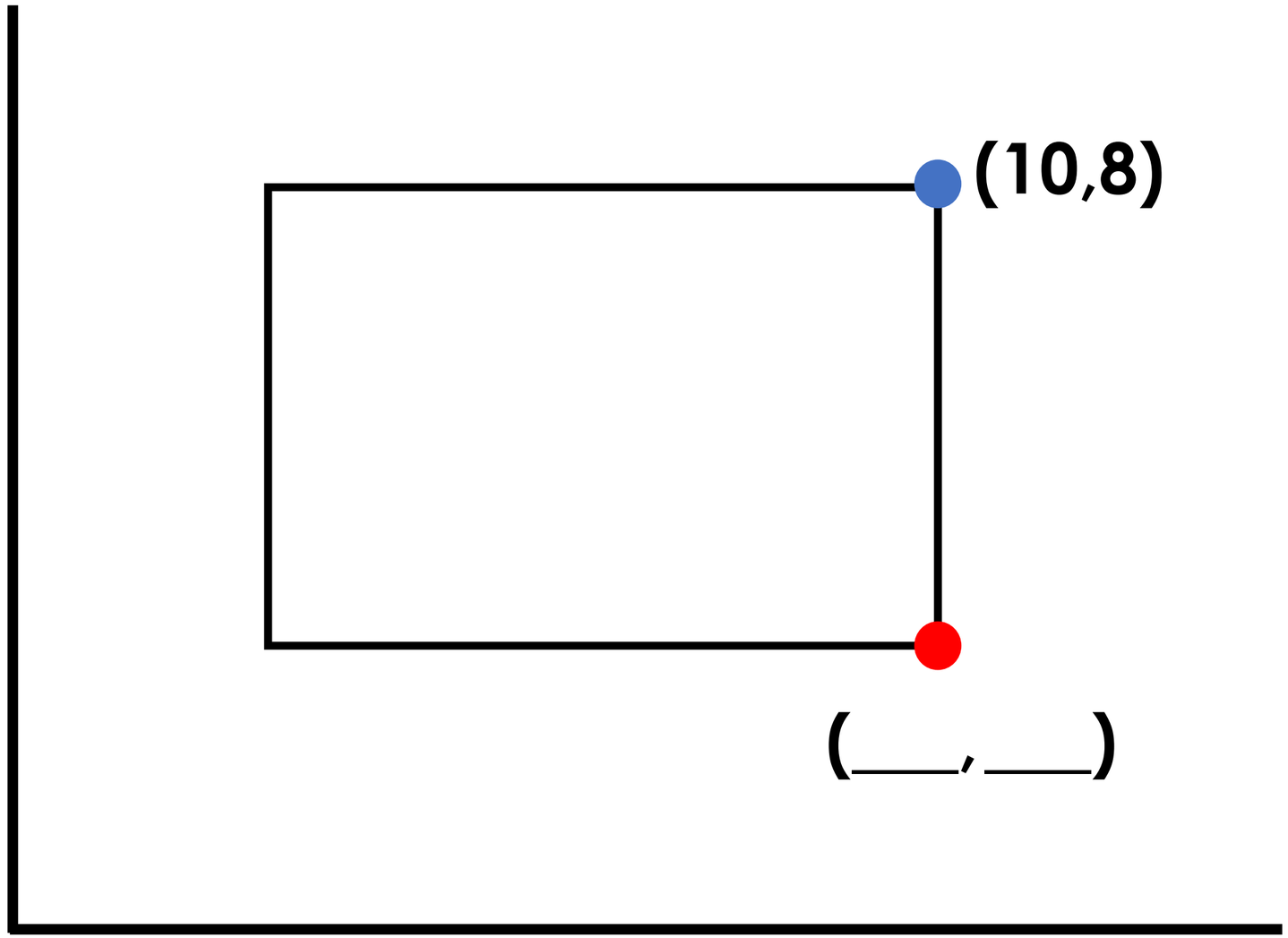
One calculation

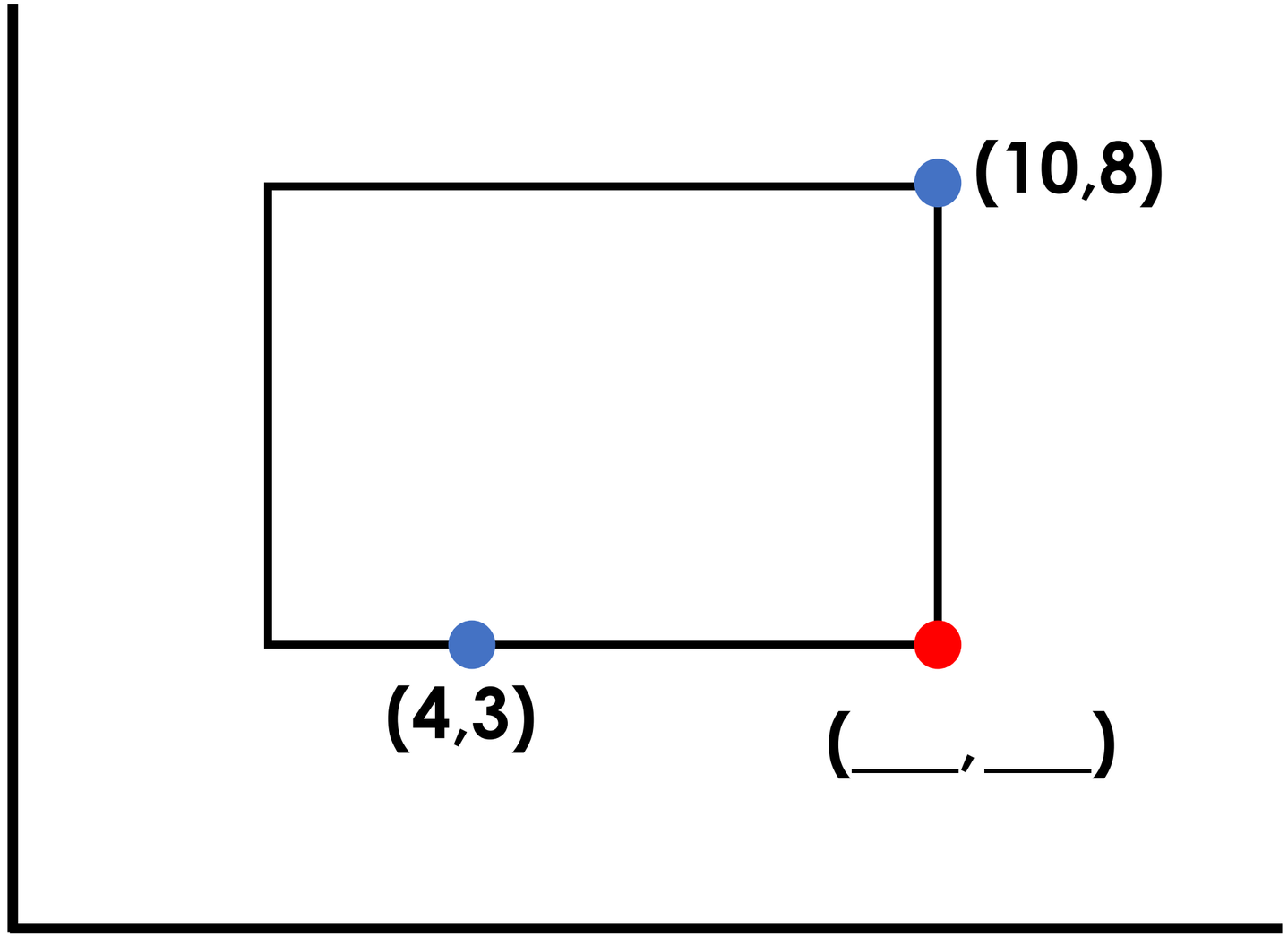
Multi-step calculation

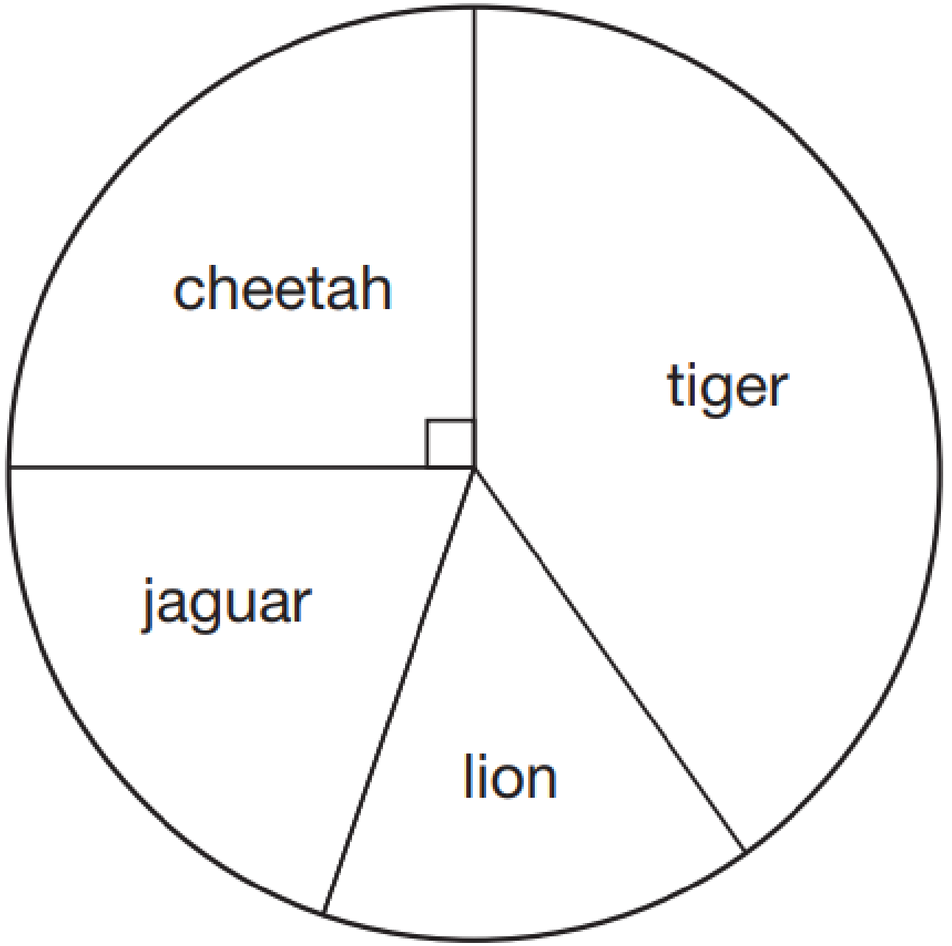

(10,8)
(4,3)
(____, ____)



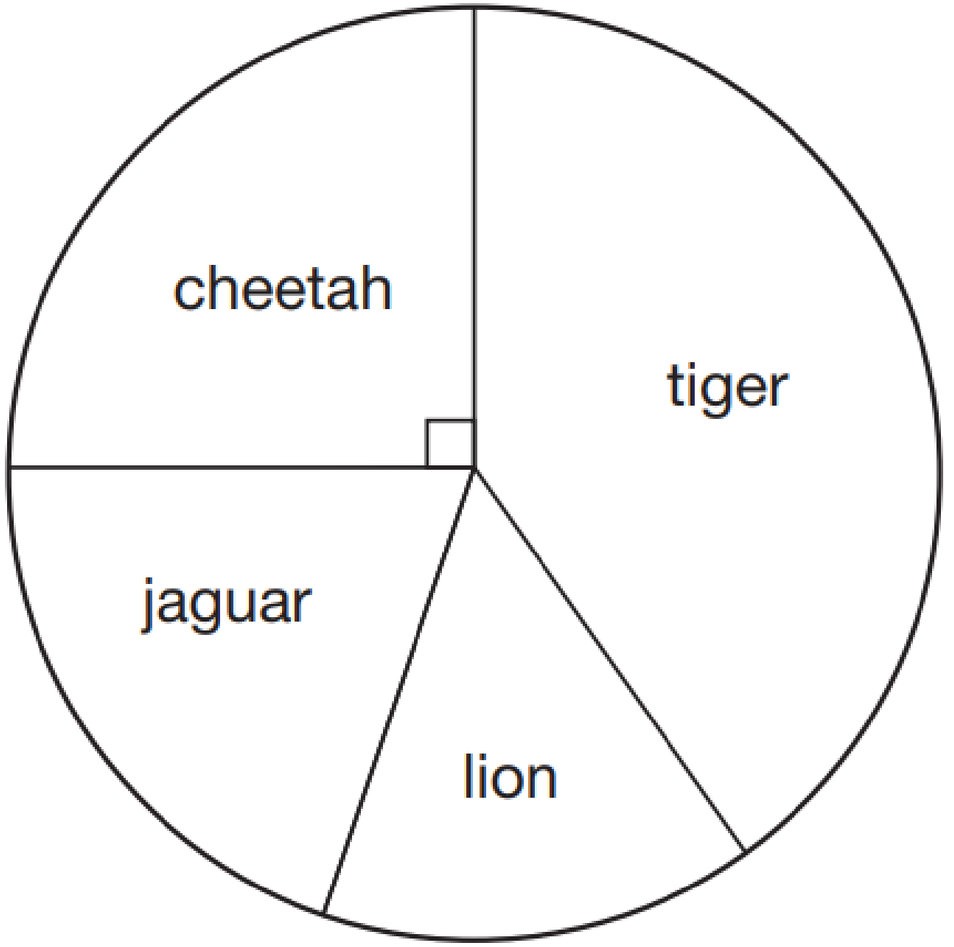



(10,8)
(____, ____)


(10,8)
(4,3)
(____, ____)


cheetah
tiger
jaguar
lion

There are ☐ big cats in the zoo altogether.



Here are some statements about the chart.

Tick the statements that are **true**.

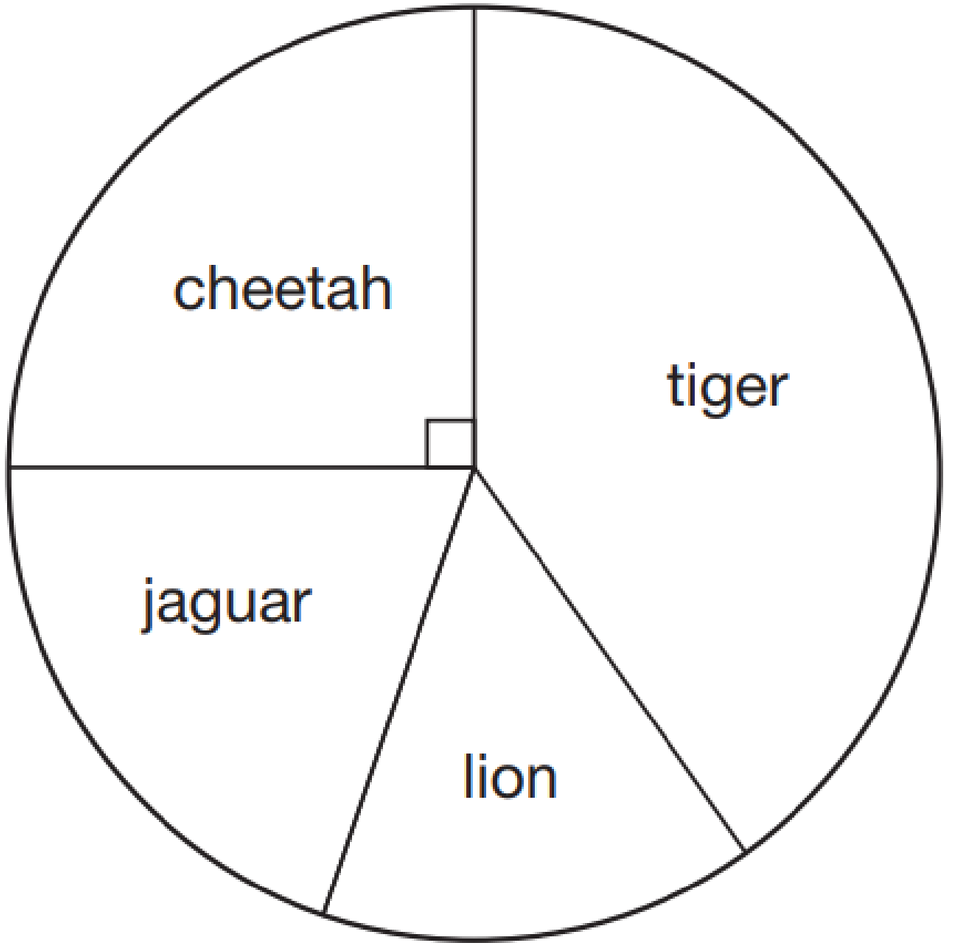
☐

☐

☐

☐

There are ☐ big cats in the zoo altogether.



Here are some statements about the chart.

Tick the statements that are **true**.

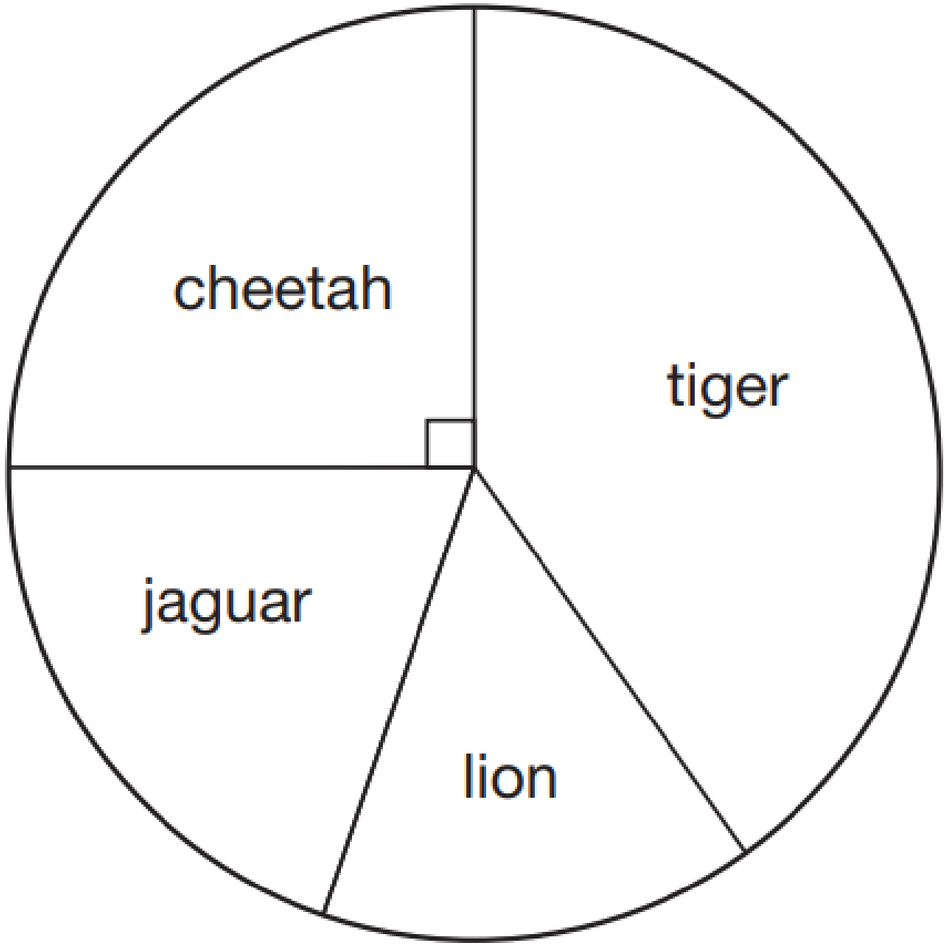
There are more cheetahs than jaguars. ☐

The total number of lions and tigers is 10 ☐

One-quarter of the big cats are cheetahs. ☐

There are more than 5 jaguars. ☐

There are ☐ big cats in the zoo altogether.

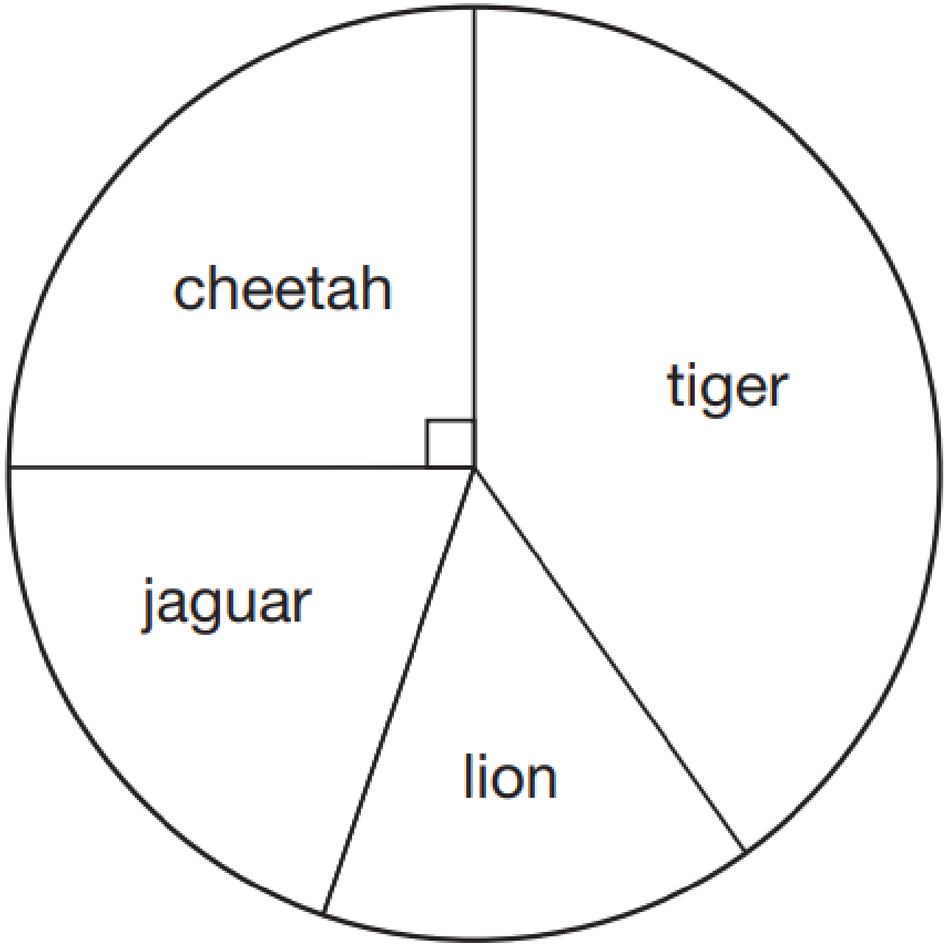


Here are some statements about the chart.

Tick the statements that are **true**.

There are more cheetahs than jaguars.	✓
The total number of lions and tigers is 10	
One-quarter of the big cats are cheetahs.	✓
There are more than 5 jaguars.	

There are **20** big cats in the zoo altogether.



Here are some statements about the chart.

Tick the statements that are **true**.

There are more cheetahs than jaguars.	✓
The total number of lions and tigers is 10	
One-quarter of the big cats are cheetahs.	✓
There are more than 5 jaguars.	

There are 28 pupils in a class.

The teacher has 8 litres of orange juice.

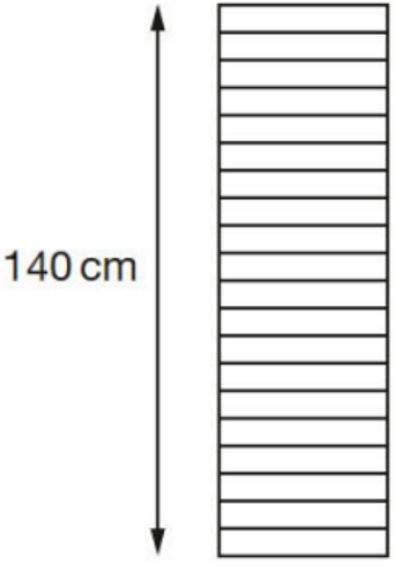
She pours 225 millilitres of orange juice for every pupil.

How much orange juice is left over?

A stack of 20 identical boxes is 140cm tall.

Stefan takes **three** boxes off the top.

How tall is the stack now?



How Many Ways?

You have a pile of **0.1** and **0.01** counters.

Question 1:
How many ways can **0.42** be made?

How Many Ways?

You have a pile of **0.1** and **0.01** counters.

Question 1:
How many ways can **0.42** be made?

Question 2:
How many ways can **0.24** be made?

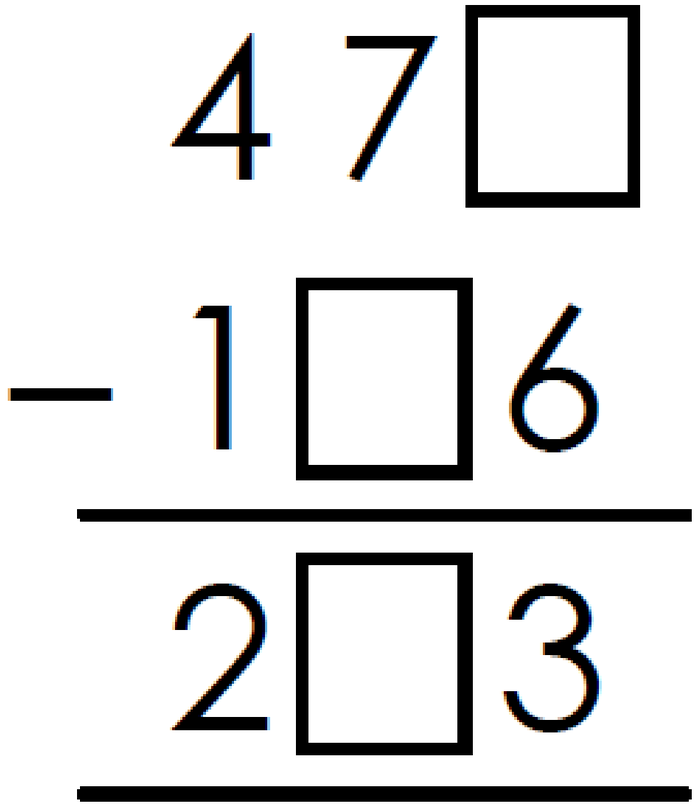
How Many Ways?

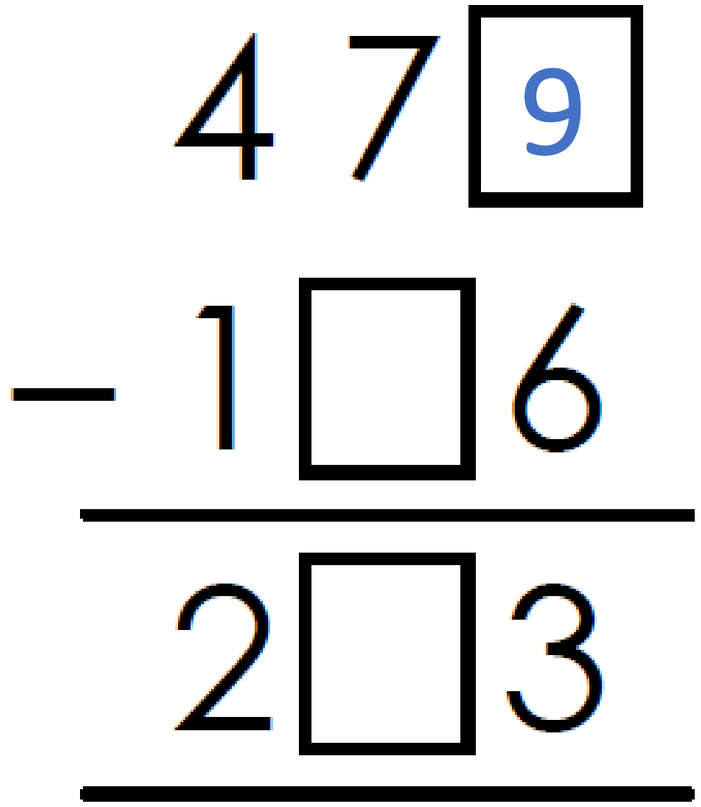
You have a pile of **0.1** and **0.01** counters.

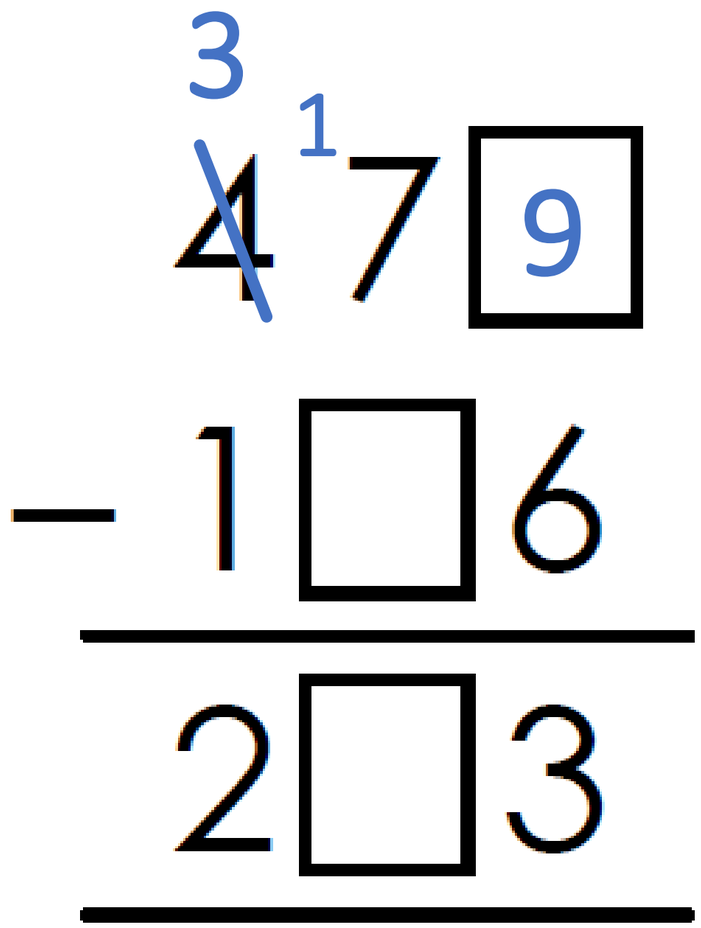
Question 1:
How many ways can **0.42** be made?

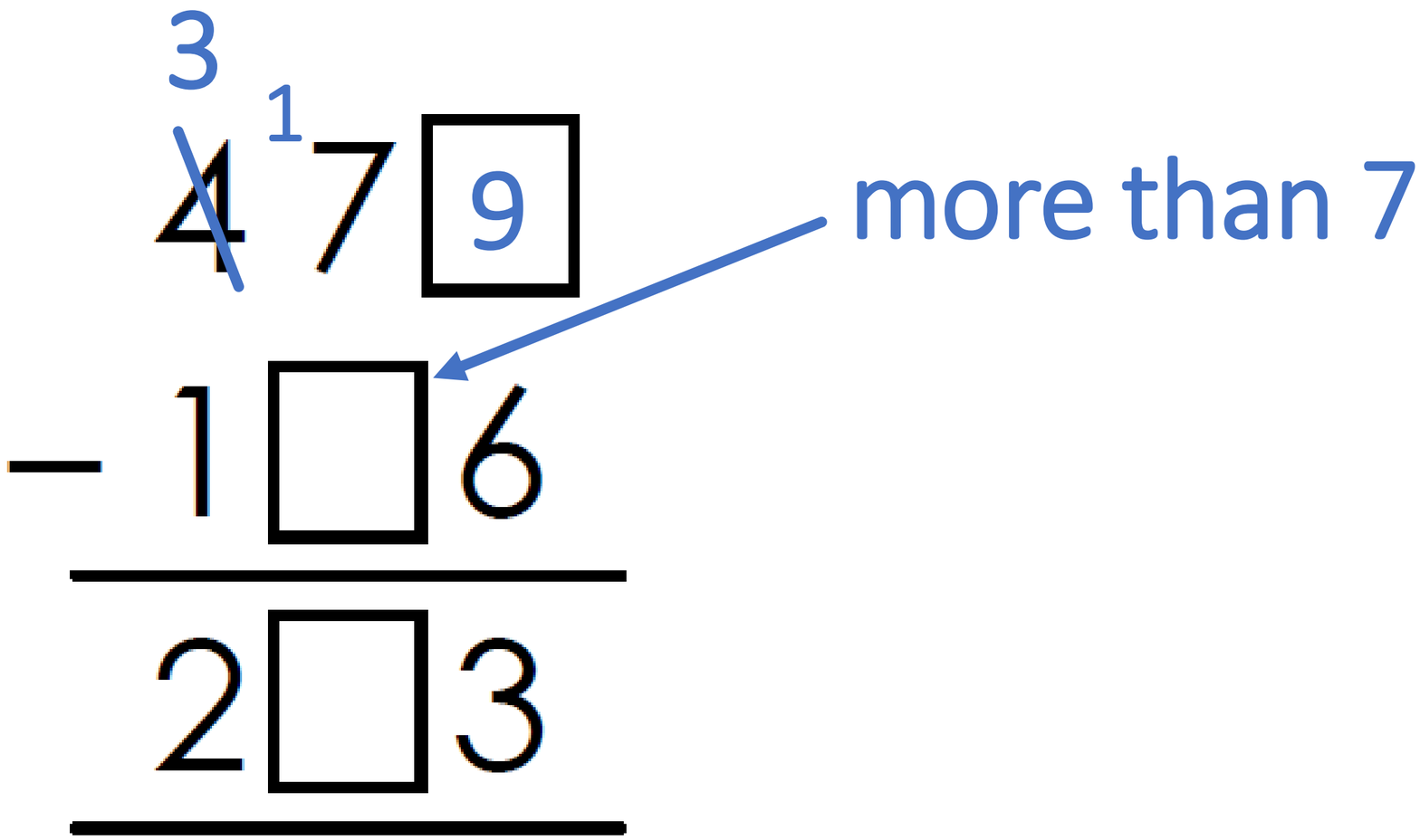
Question 2:
How many ways can **0.24** be made?

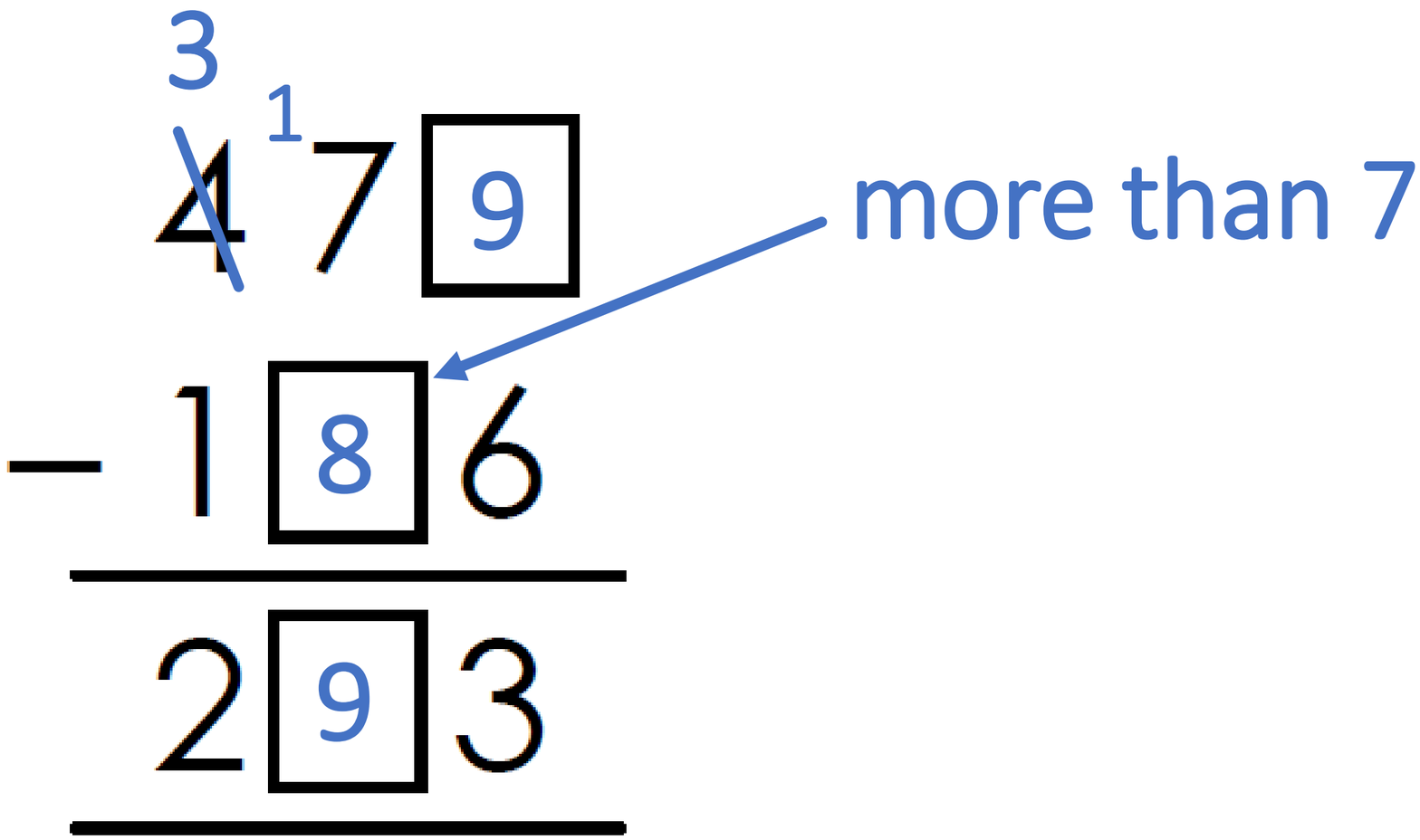
Agree or Disagree:
‘0.35 can be made in more ways than 0.32’

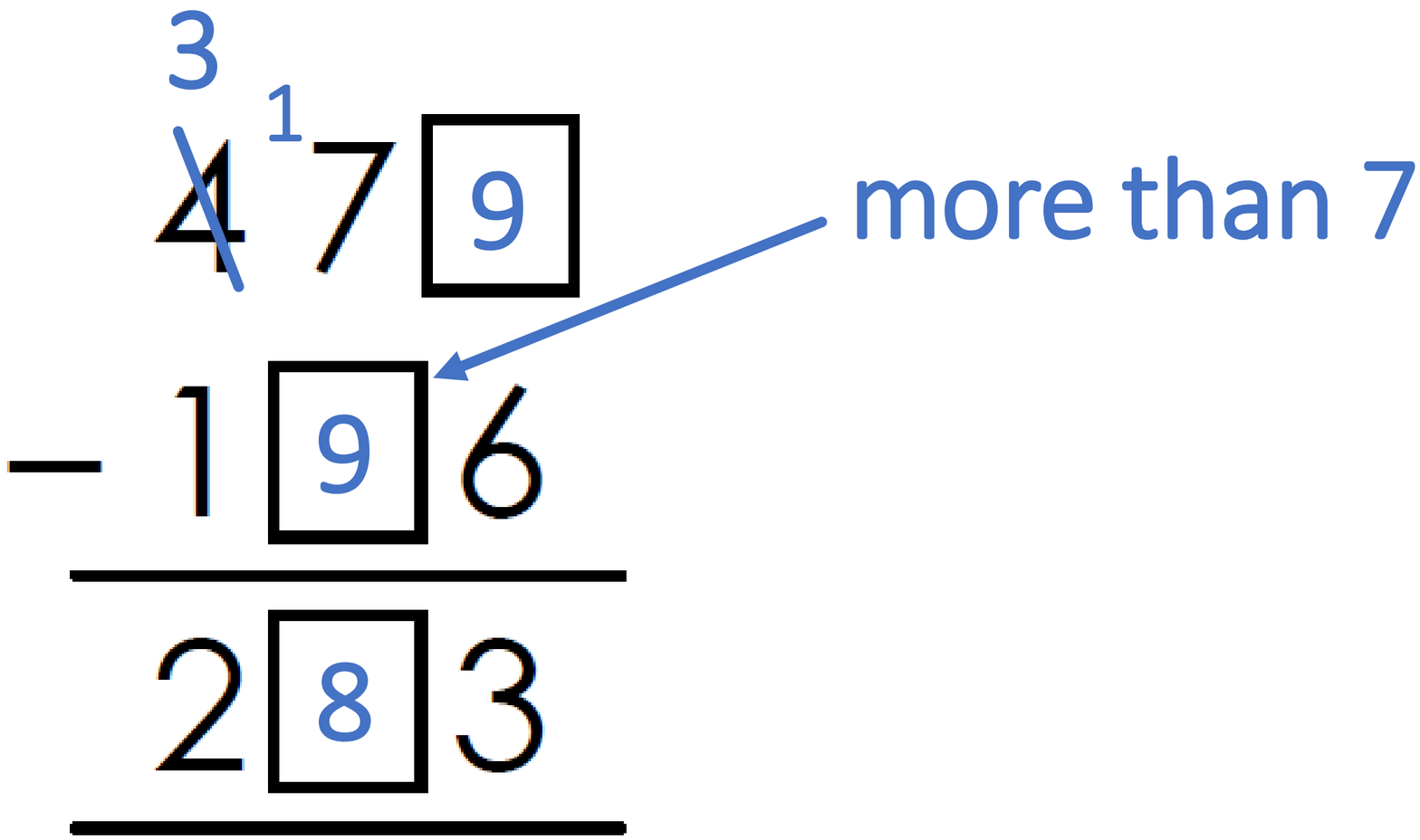

4 7
– 1 6
2 3


4 7 9
– 1 6
2 3


3
1
4 7 9
– 1 6
2 3


3
4 1 7 9
more than 7
– 1 6
2 3


3
1
4 7 9
more than 7
– 1 8 6
2 9 3


3
1
4 7 9
more than 7
– 1 9 6
2 8 3

126 sum of the digits: **1 + 2 + 6 = 9**

76 sum of the digits: **7 + 6 = 13**

Investigate

Make the two numbers using digits 0-9 (no repeats).

Make the difference between the numbers as small as possible.

Investigate

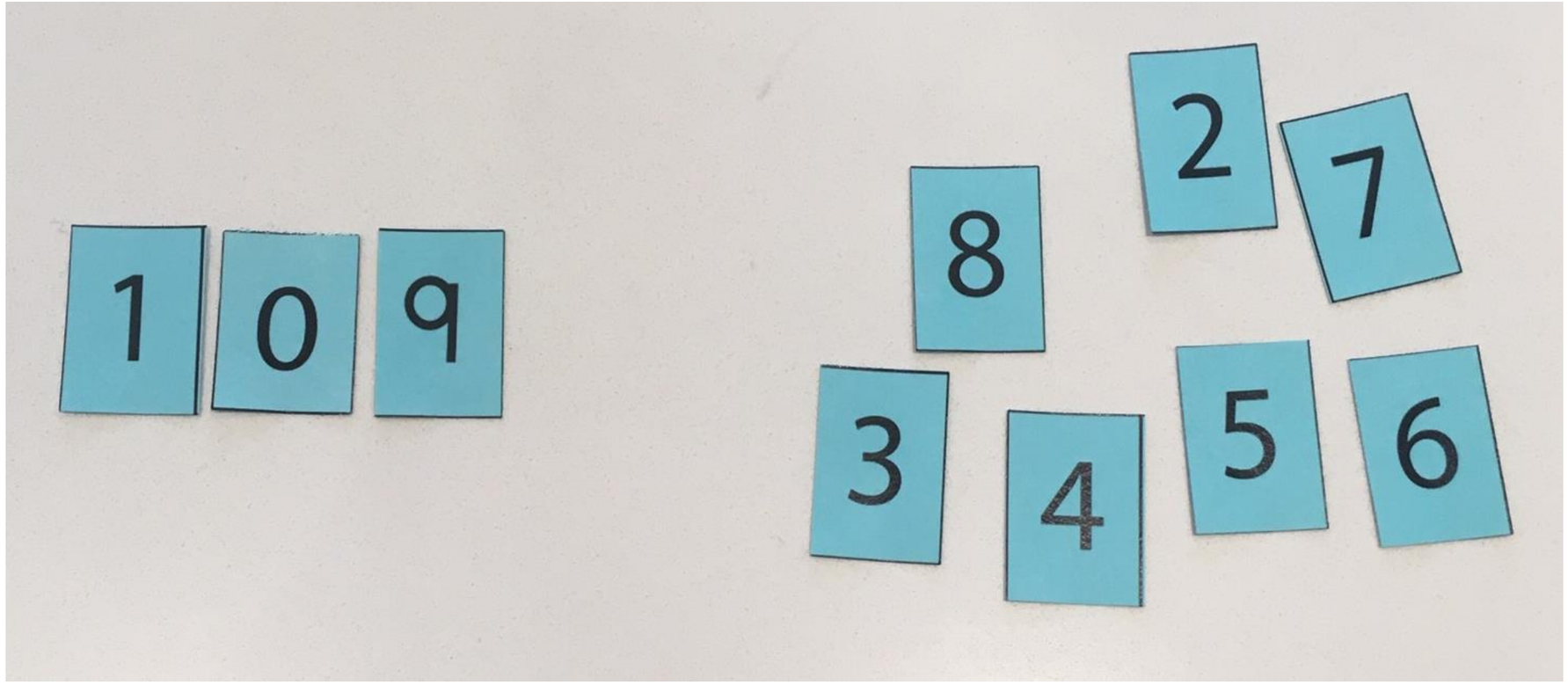
The sum of the digits of a 3-digit number is larger than the sum of the digits for a 2-digit number.

Make the two numbers using digits 0-9 (no repeats).

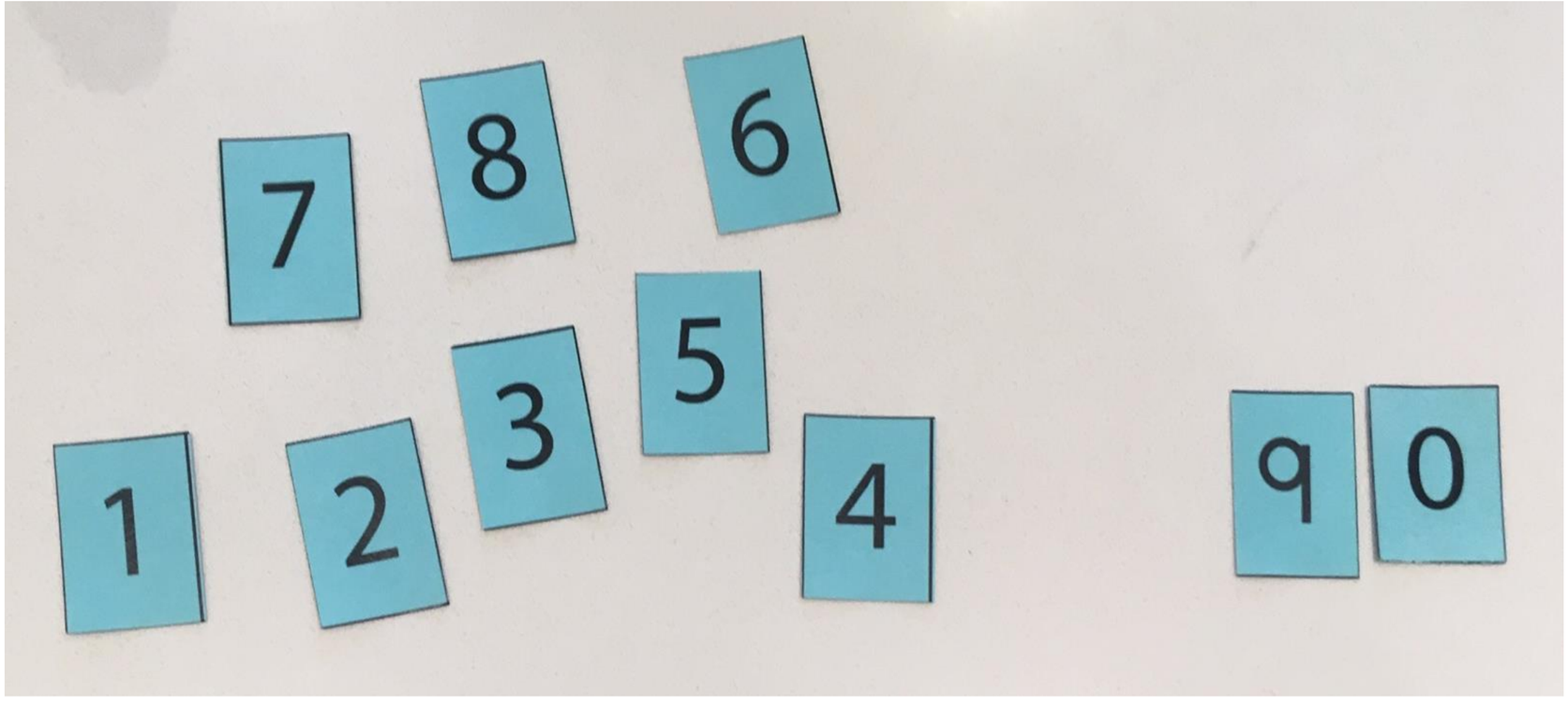
Make the difference between the numbers as small as possible.


102
98


102
98


109
8
2
7
3
4
5
6


102
98


7
8
6
5
3
1
2
4
90