

Creating Reasoning Routines, Building Problem-Solvers

Session 2

***Reasoning throughout
knowledge acquisition***

www.iseemaths.com

Y3 & Y4



Routines Within Interactive Teaching

Raising the internal narrative:

Gap between question and response/discussion, silent modelling

Mass participation:

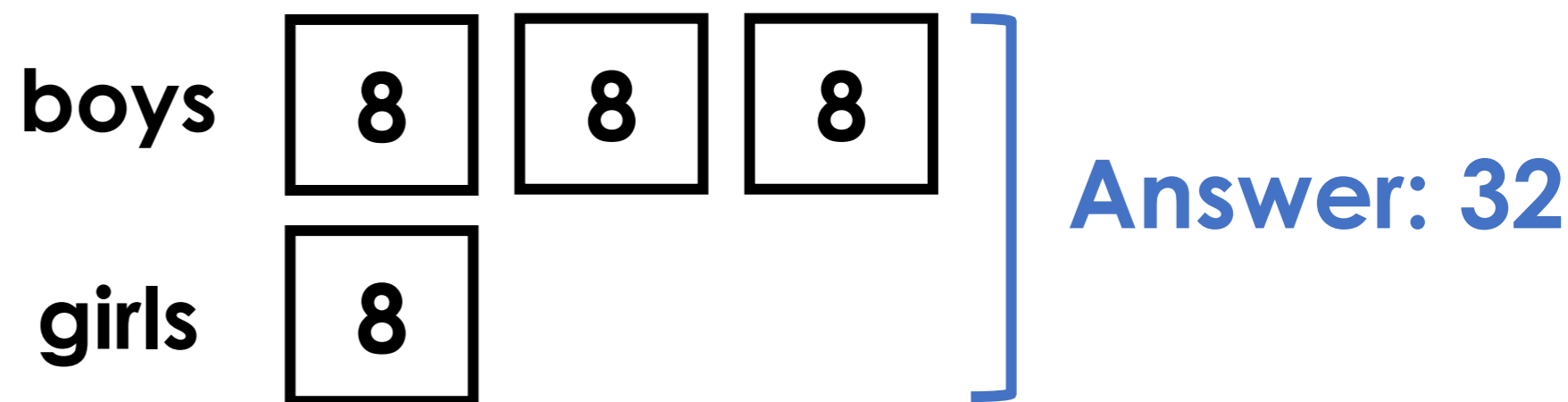
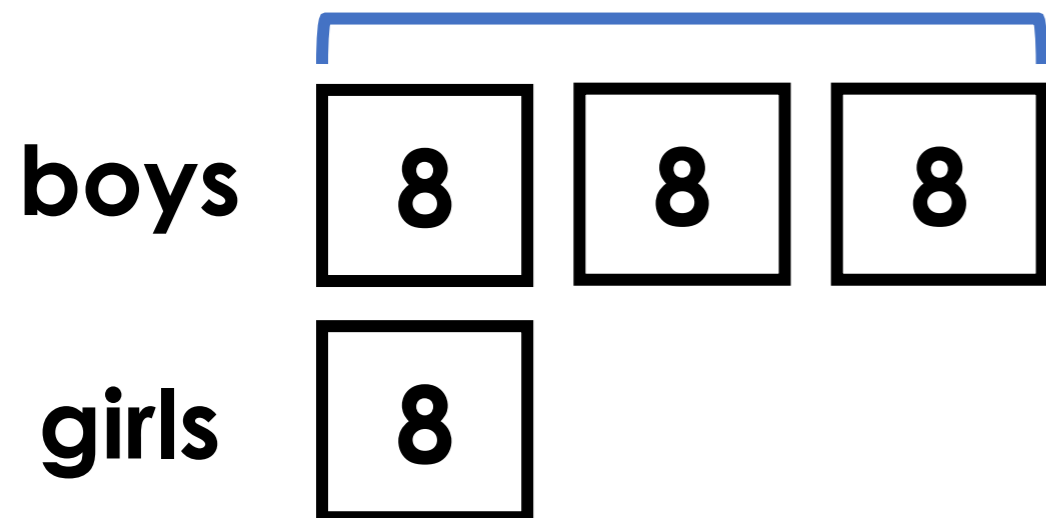
Form of answer before question, routines

Managing discussions:

Quality modelling + participation, wait time 2

There are 8 girls at the party. There are 3 times as many boys as girls at the party. **How many children at the party?**

Answer: 24

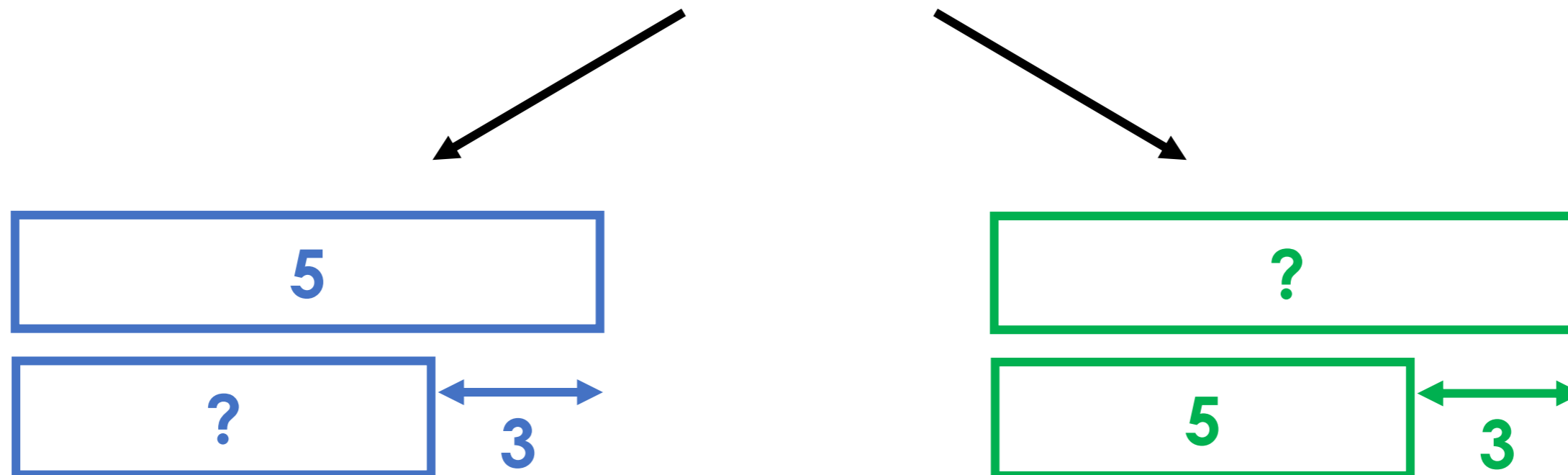


Jen collected 5 shells.

Jen collected 3 shells fewer than Zara.

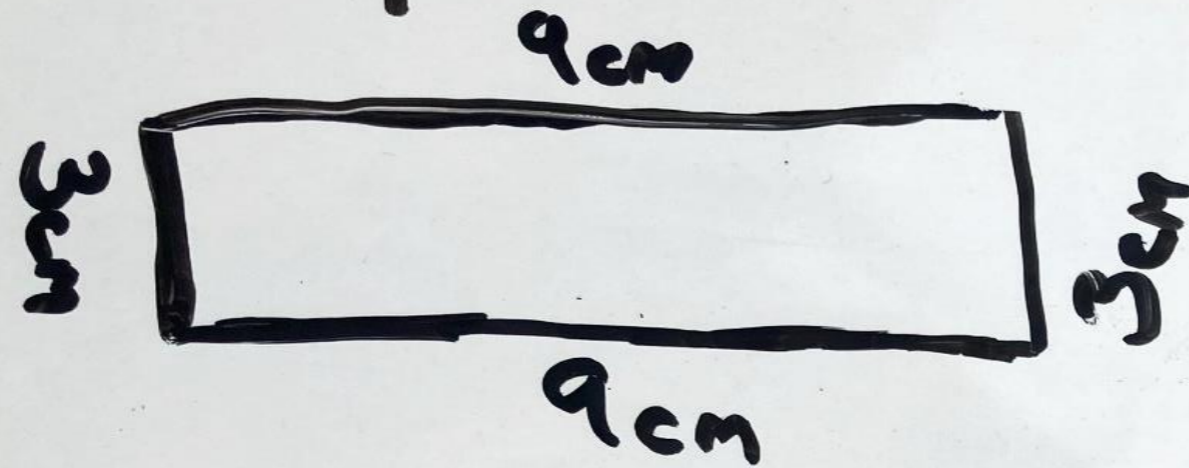
How many shells did Zara collect?

Which bar model represents the question?



A rectangle with a perimeter of 24cm.

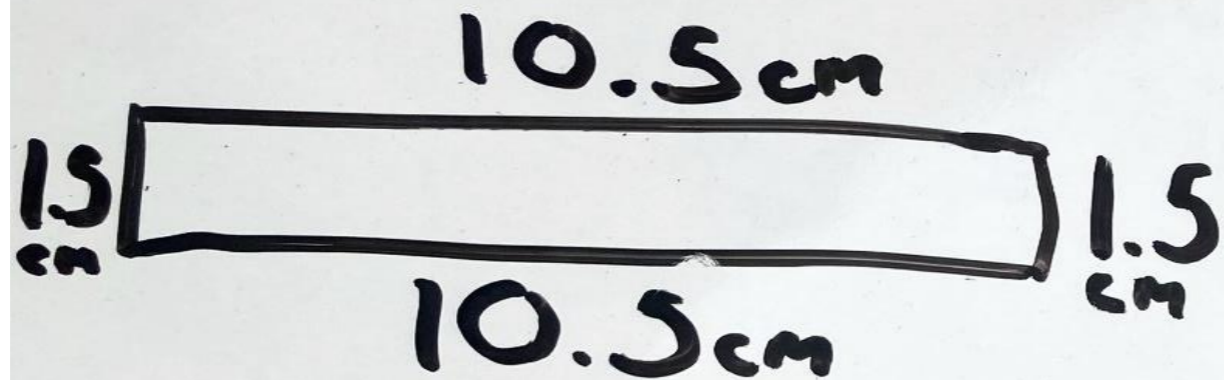
Example 1



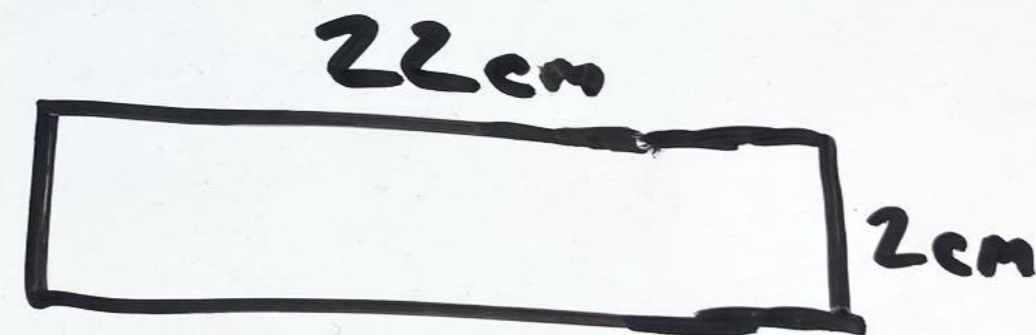
Example 2



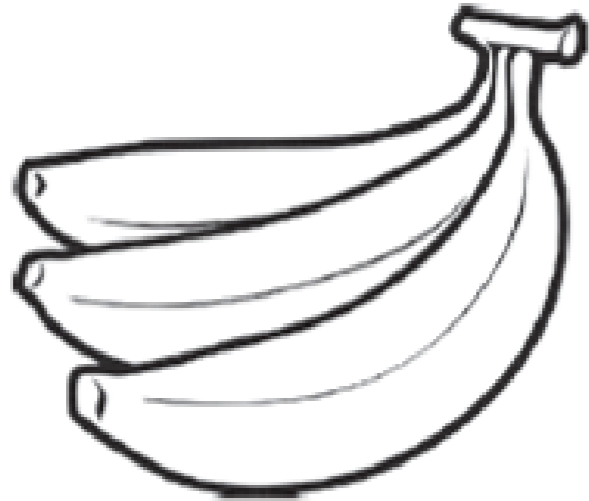
Interesting example



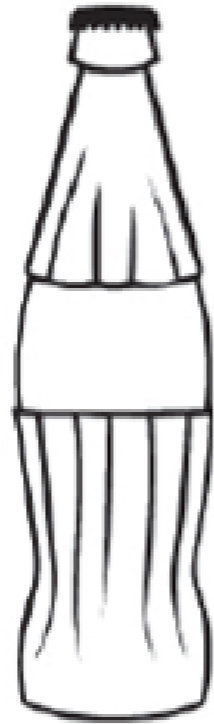
Non-example



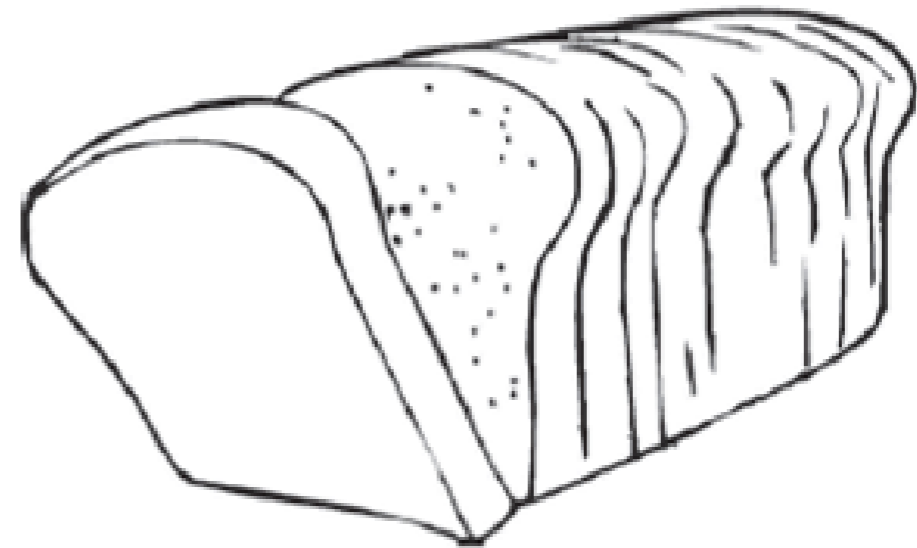
Here is the cost of some items in a shop.



Banana 20p



Drink 50p



Bread 40p



How many bananas can he buy?

Jen has 10 sweets.



How many sweets do they have altogether?

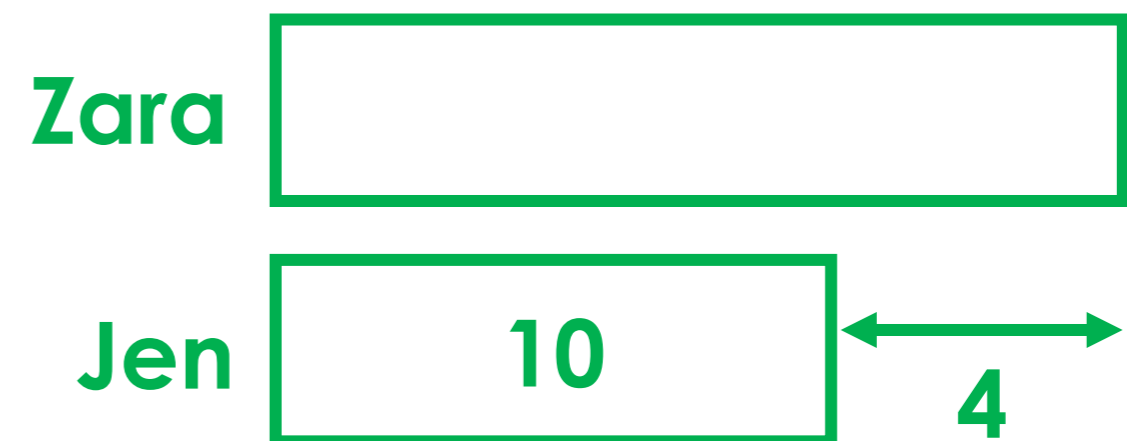
Jen has 10 sweets.

Zara has 4 more sweets than Jen.

How many sweets do they have altogether?

Explain the mistake:

$$10 + 4 = 14 \text{ sweets}$$



Whitney has £30 pocket money.

She buys some of these books and gets £6 change.



How many books does she buy?

Different Question Types

Rulers = 60p

Pencils = 50p

Rubbers = 30p

(a) Tim buys two rulers and a pencil.

How much does it cost?

(b) Jen has £2. **How many rulers can she buy?**

(c) Kam buys two pencils and a rubber. He pays £2.

How much change does he get?

(d) Zara wants two rulers and a pencil. She has £1.50.

How much more money does she need?

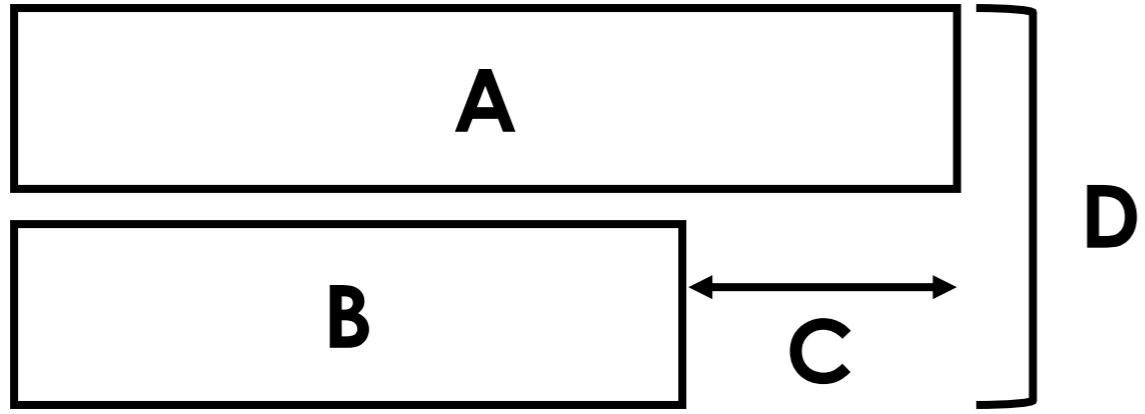
Contexts

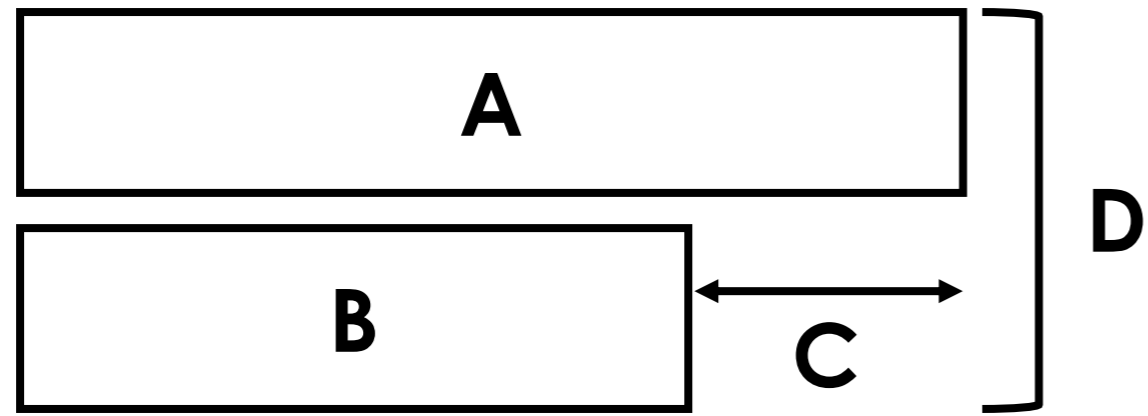
Which questions are answered by calculating $15 \div 5$?

- (a) There are 15 children in the hall. There are 5 children at each table. **How many tables are there in the hall?**
- (b) 15 people have 5 grapes each. **How many grapes in total?**
- (c) 5 children share 15 pencils. **How many pencils each?**
- (d) Of the 15 children that go to running club, there are 5 girls. **How many boys go to running club?**

Interpreting Remainders

- (a) Cupcakes are packed in boxes of 6.
The bakery has made 30 cupcakes.
How many full boxes can be made?
- (b) Cupcakes are packed in boxes of 6.
The bakery has made 32 cupcakes.
How many full boxes can be made?
- (c) Cupcakes are packed in boxes of 6.
The bakery has made 32 cupcakes.
How many boxes are needed to hold all of the cupcakes?





1. Raja has 6 stickers. He has 4 stickers less than Zack.

How many stickers does Zack have?

2. Jen has 6 stickers. Helen has 4 stickers. In total, they have 10 stickers.

How many more stickers does Jen have than Helen?

3. Tim has 6 stickers. He has 4 more stickers than Mo.

How many stickers do Tim and Mo have in total?

There are **8** children at the playground.

There are more boys than girls.

There are **8** children at the playground.

There are **2** more boys than girls.

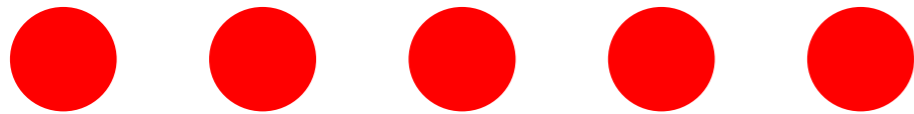
There are **8** children at the playground.

There are **2** more boys than girls.



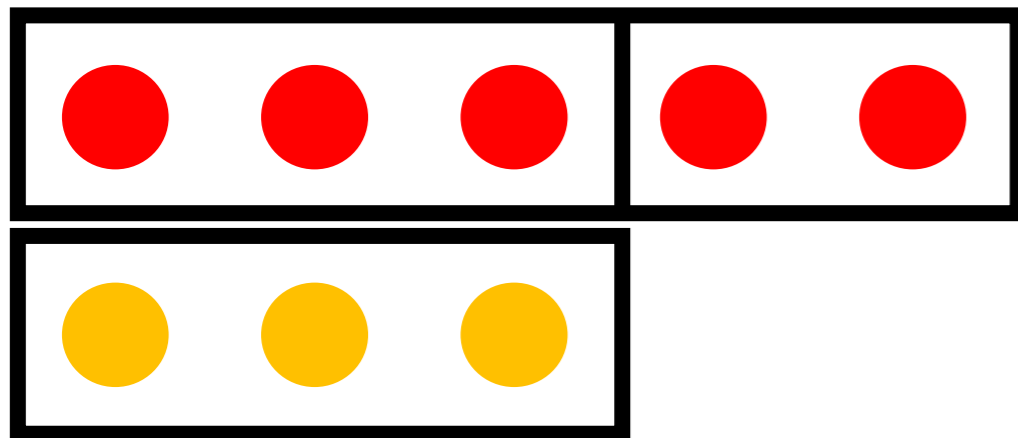
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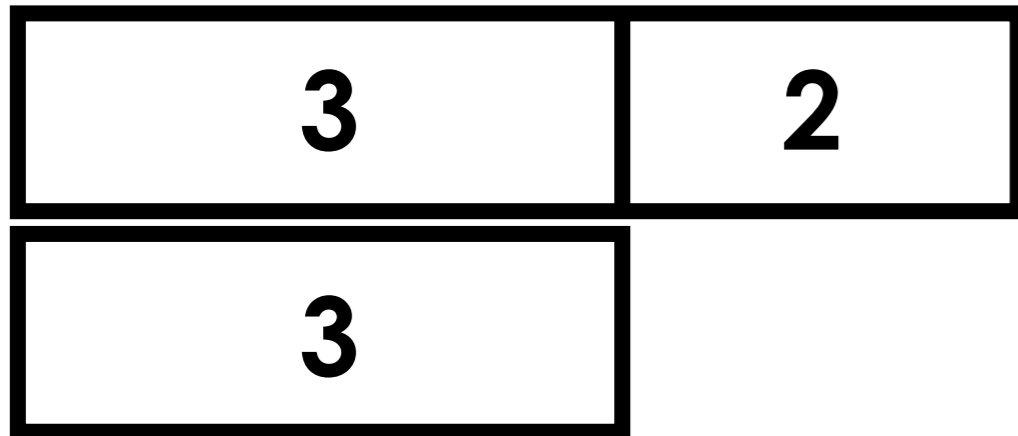
There are **8** children at the playground.

There are **2** more boys than girls.



There are **8** children at the playground.

There are **2** more boys than girls.



T
A
S
K
1

There are 8 children at the park.

There are 2 more boys than girls at the park.

Talk: how many girls could be at the park?

Next step: your teacher will tell you the number in the red box.

Answer: There are 3 girls at the park.

T
A
S
K
2

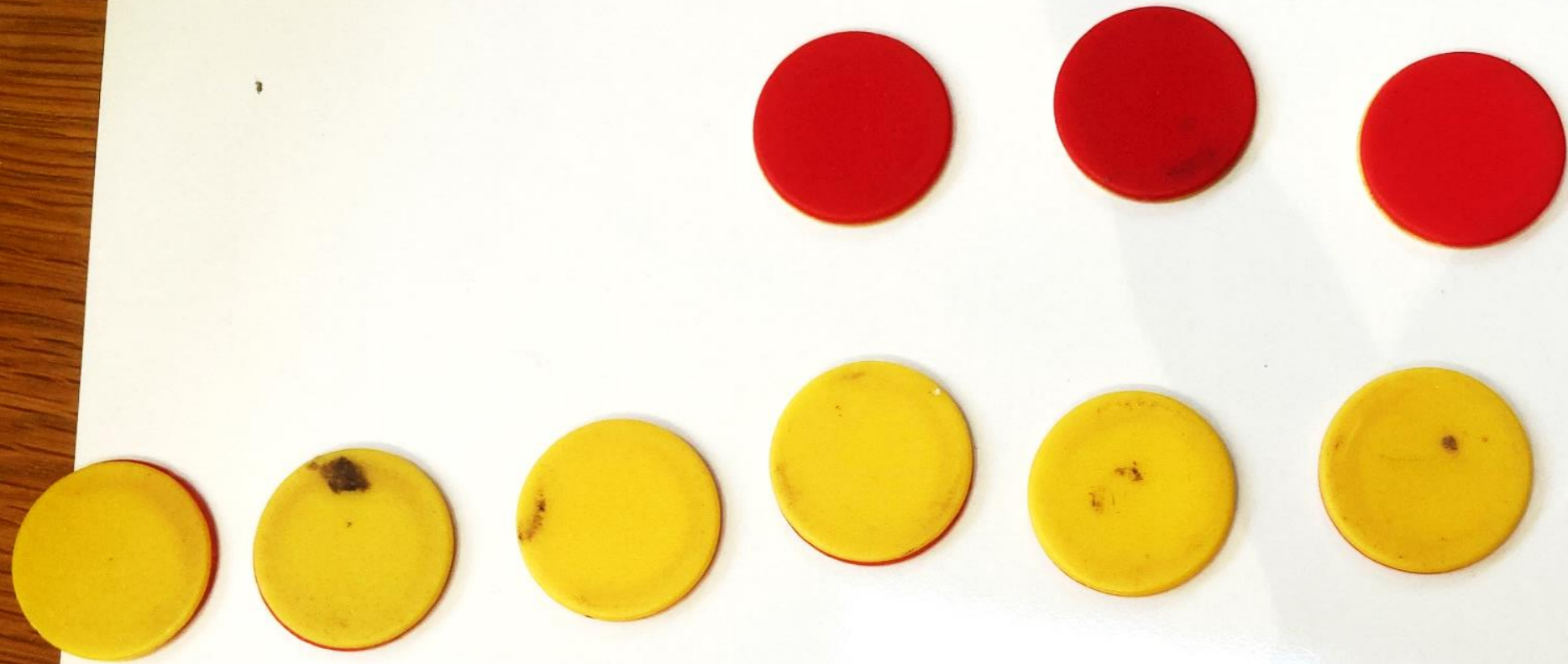
There are 14 children at the party.

There are 4 more girls than boys at the party.

Talk: how many boys could be at the party?

Next step: your teacher will tell you the number in the red box.

Answer: There are 5 boys at the party.

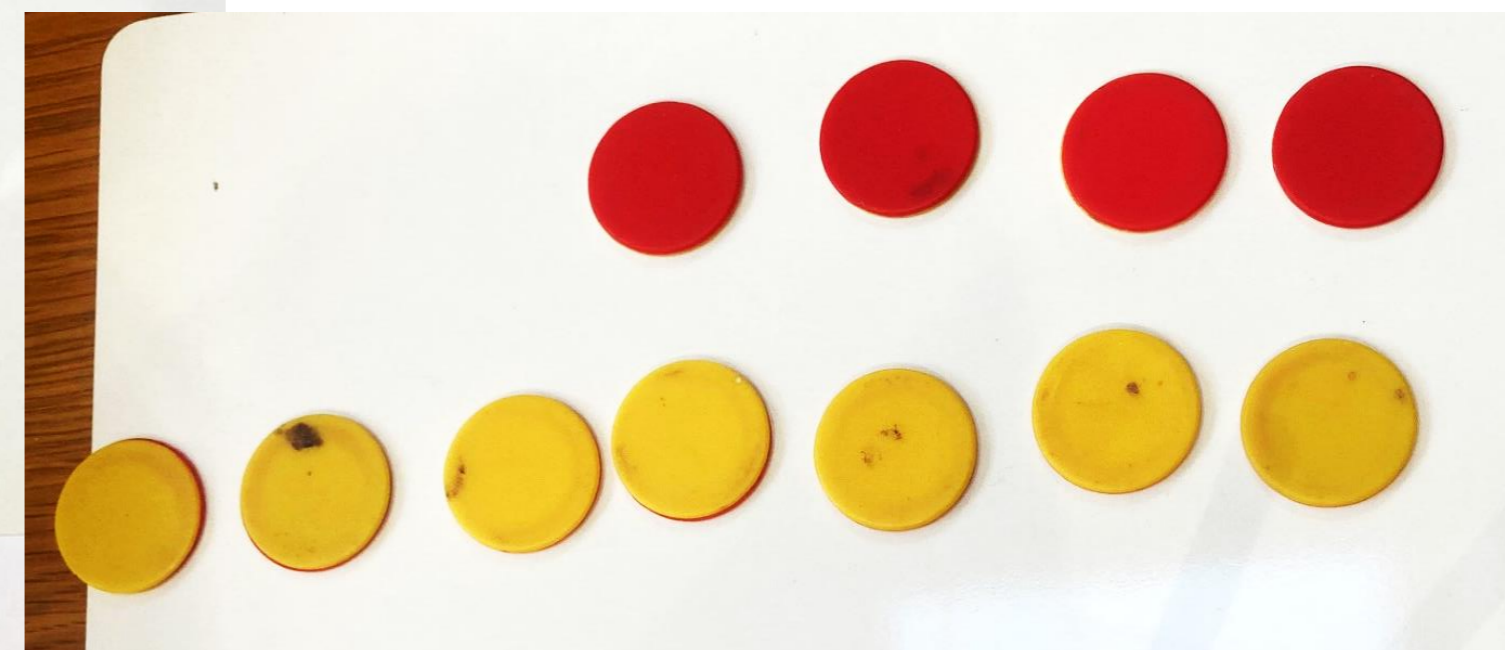
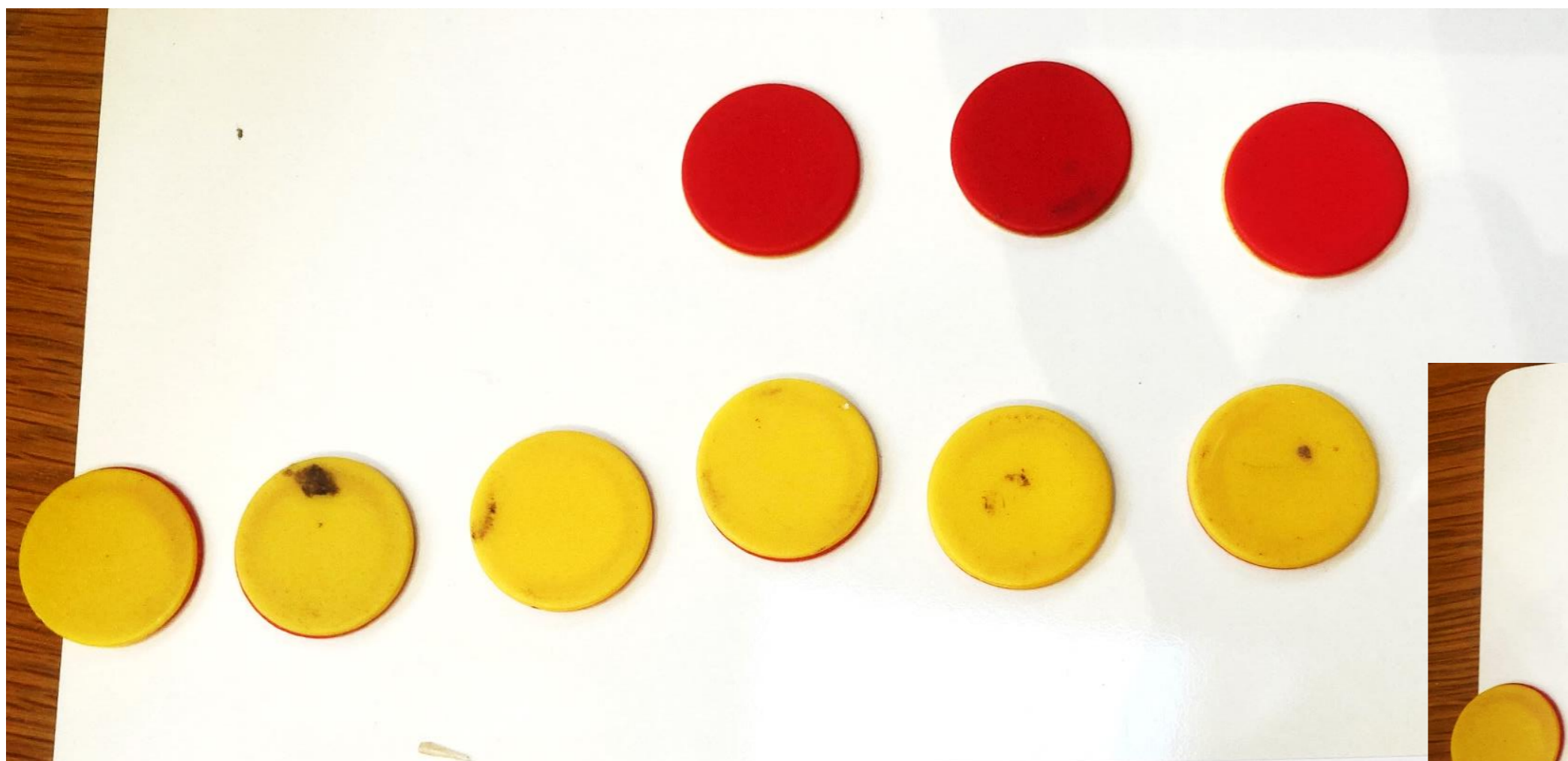


Question 1: There are 9 children at the party. There are 7 boys.
How many girls are at the party? 3

Question 2: There are 9 children at the park. There are more girls than boys. **How many boys could be at the park?** ~~7 8 6 5 4 3 2 1~~ 4 0 1 2

Question 3: There are 9 children at the park. There are 3 more girls than boys. **How many girls are at the park?**

Question 4: There are 11 children at the party. There are 3 more girls than boys. **How many girls are at the party?**



Question 1: There are 9 children at the party. There are 7 boys.
How many girls are at the party? 3

Question 2: There are 9 children at the park. There are more girls than boys. **How many boys could be at the park?** ~~7 8 6 5 4 3 2 1~~ 4 0 1 2

Question 3: There are 9 children at the park. There are 3 more girls than boys. **How many girls are at the park?**

Question 4: There are 11 children at the party. There are 3 more girls than boys. **How many girls are at the party?**

There were 120 passengers on the train when it arrived at York train station.

At York, 35 people got off the train and 45 people got on.

How many passengers were on the train when it left York train station?

There were 120 passengers on the train when it arrived at York train station.



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At York, people got off the train and people got on.

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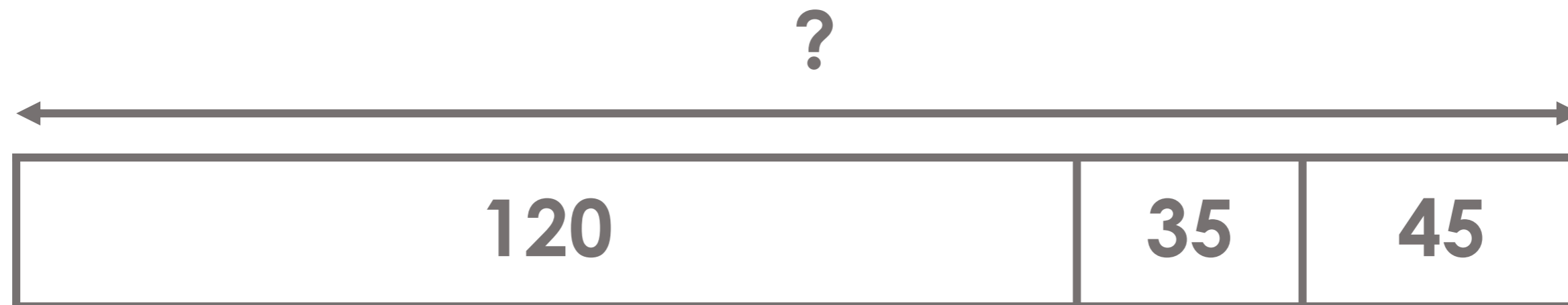
How many passengers were on the train when it left York train station?

There were 120 passengers on the train when it arrived at York train station.

At York, 35 people got off the train and 45 people got on.

How many passengers were on the train when it left York train station?

Explain why this DOES NOT correctly represent the question:

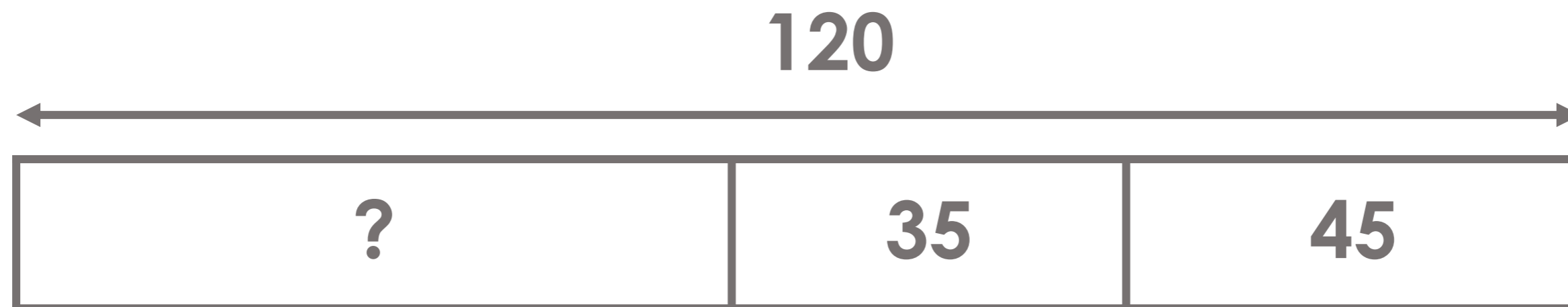


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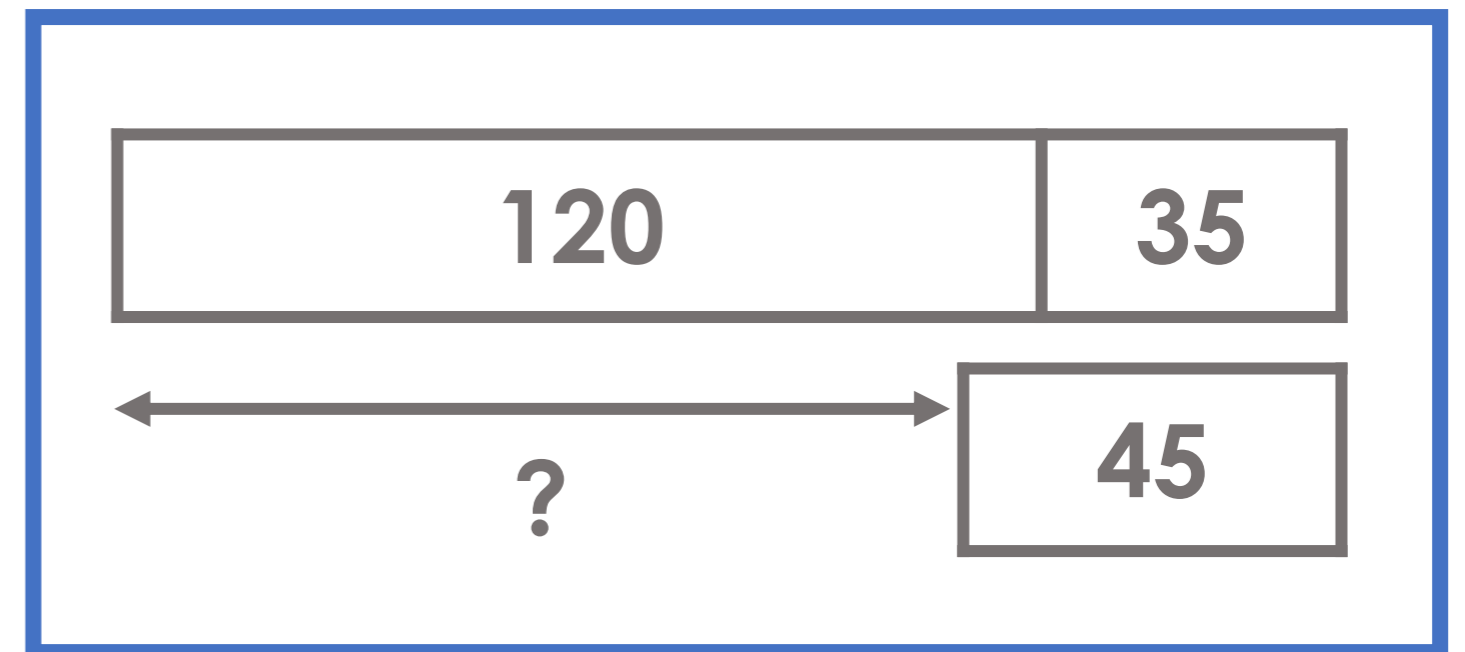
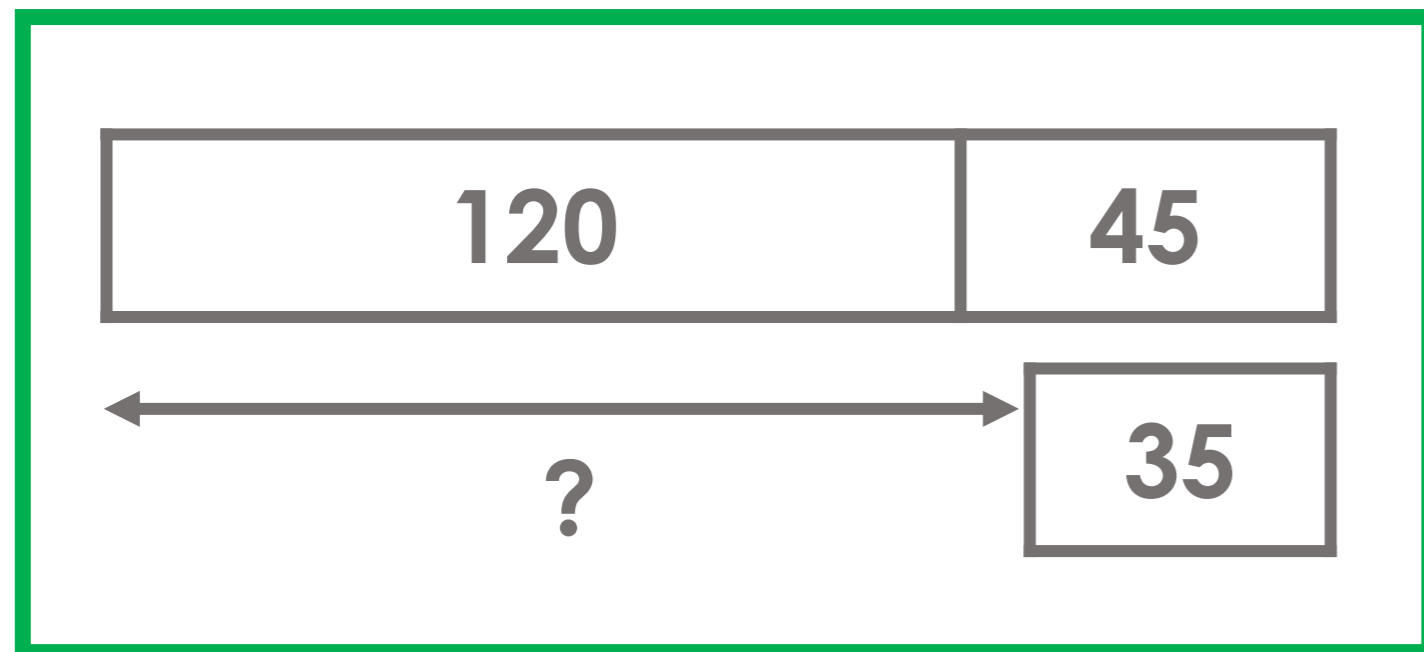


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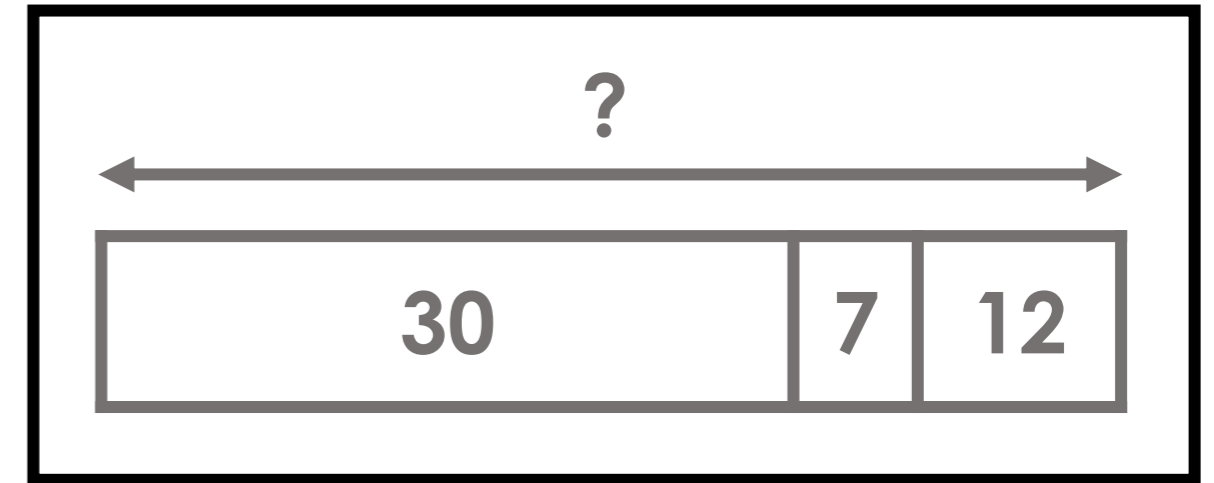
How many passengers were on the train when it left York train station?

Which Picture?



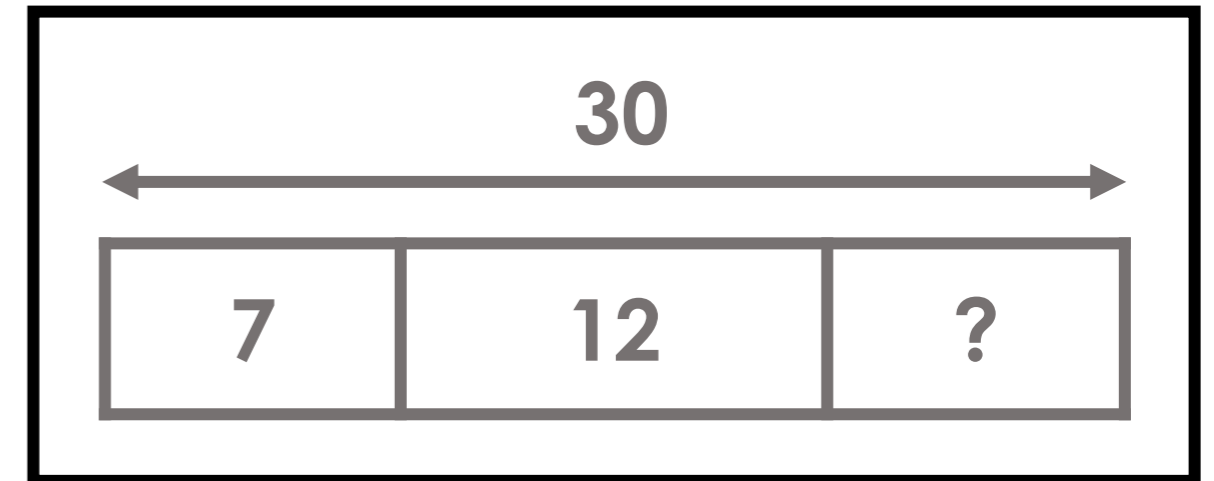
There were 30 grapes in a bowl. Lara ate 7 grapes and Jen ate 12 grapes.

How many grapes left in the bowl?



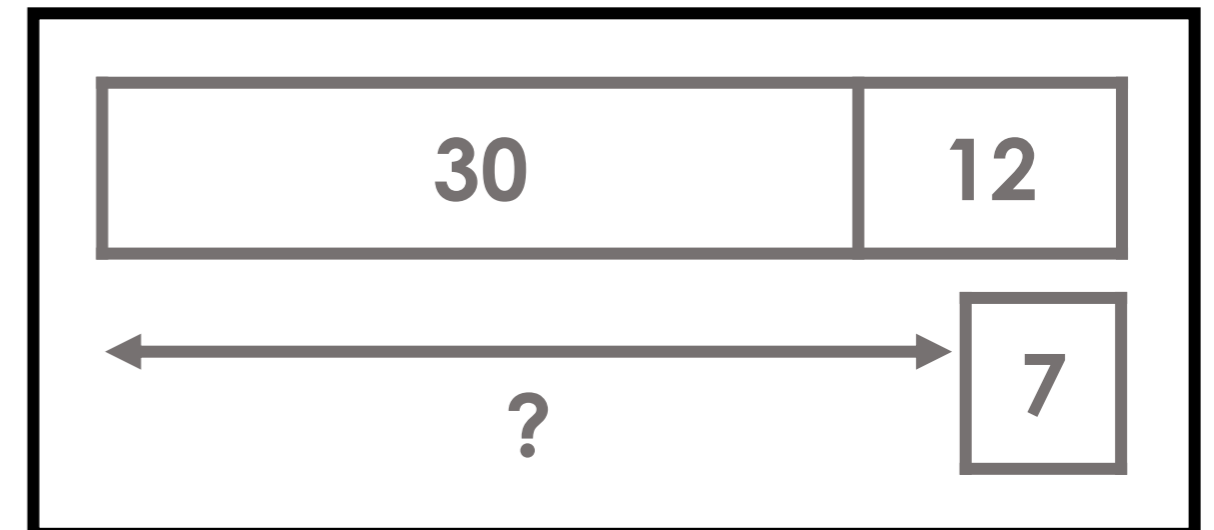
Tim had 30 stickers. He gave 7 stickers away and bought 12 more stickers.

How many stickers does he have now?

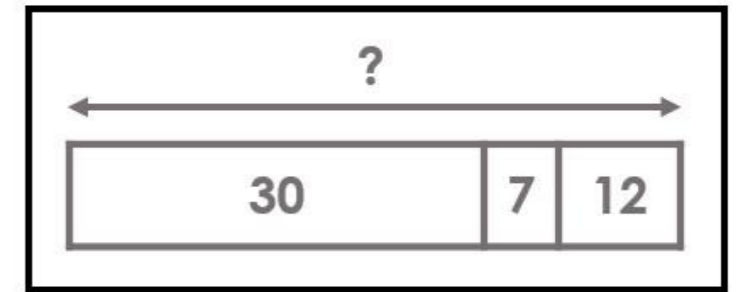


Ben has 30 sweets, Jack has 7 sweets and Jim has 12 sweets.

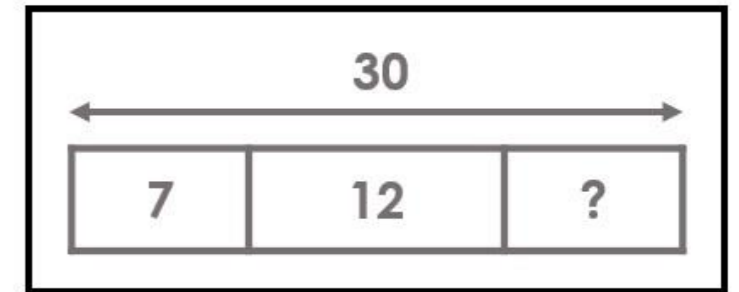
How many sweets do they have in total?



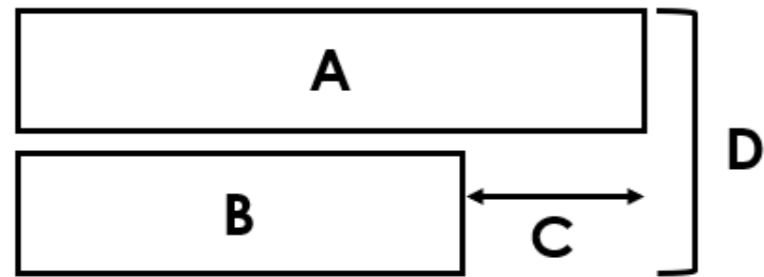
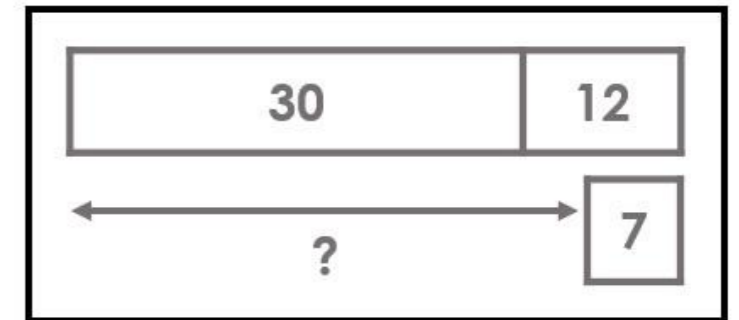
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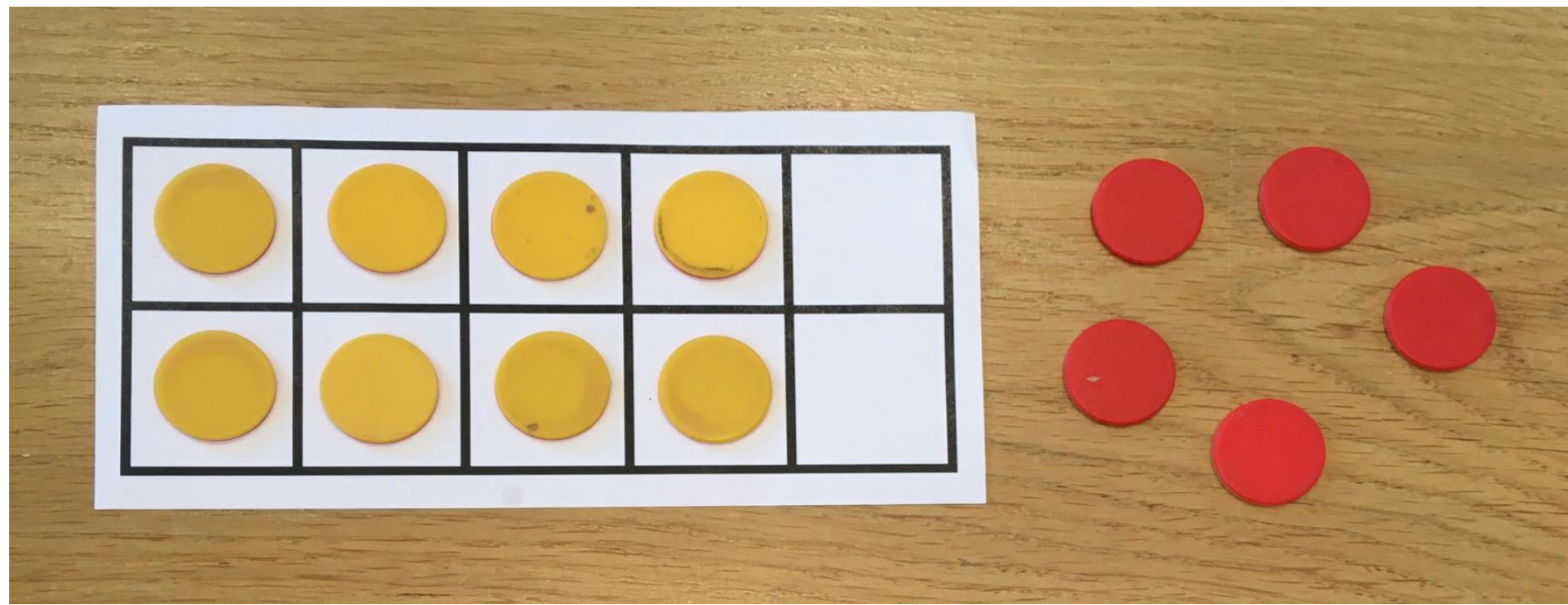
3. Tim has 6 stickers. He has 4 more stickers than Mo.

How many stickers do Tim and Mo have in total?

Reasoning Throughout Knowledge Acquisition

Using misconceptions (spot the difference, explain the mistakes, which answer?)

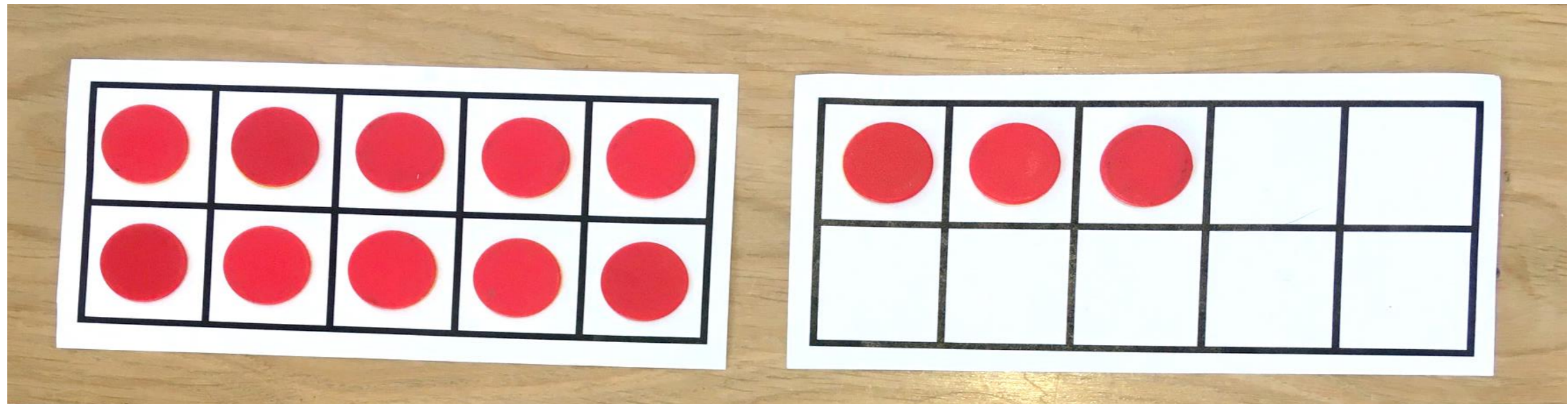
Variation, Small Difference Questions and creating own examples



$$13 - 2$$

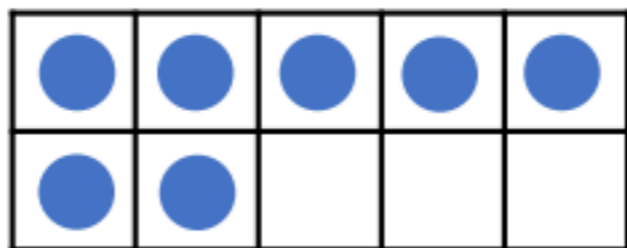
$$13 - 9$$

$$13 - 5$$



Different ways

$7 + 5 = \square$



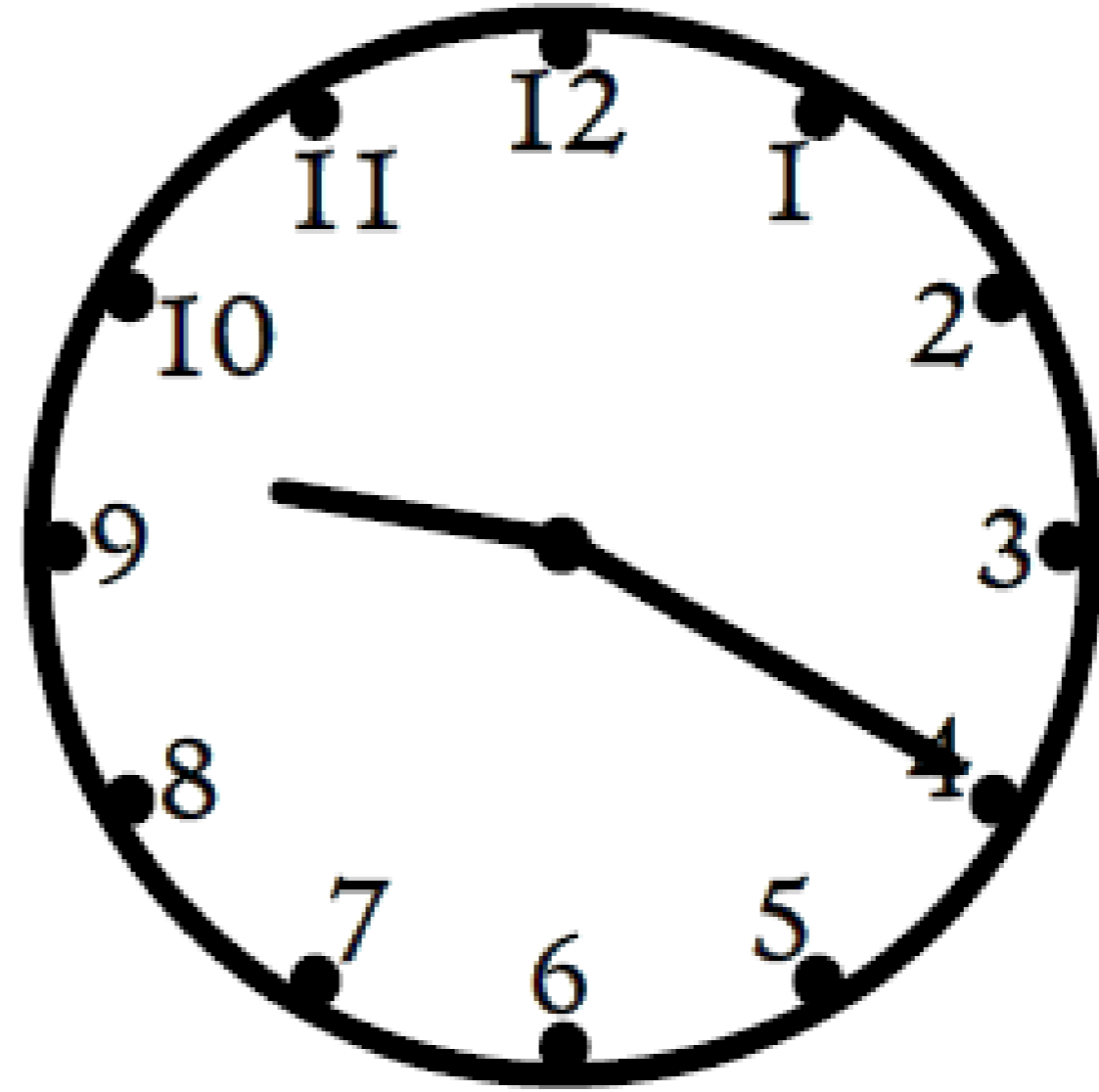
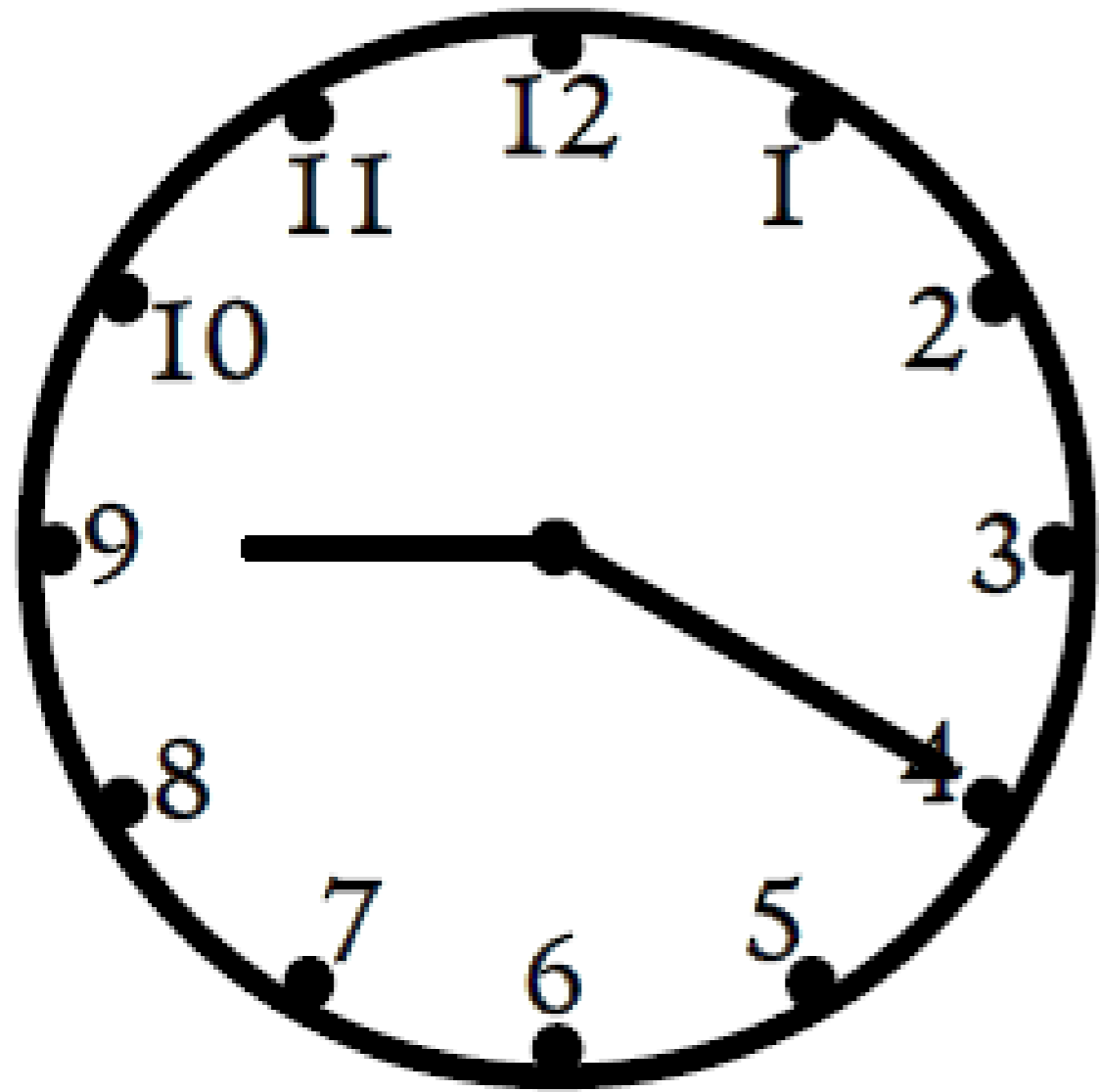
7 + 5 is the same as:

$5 + 5 + \square$

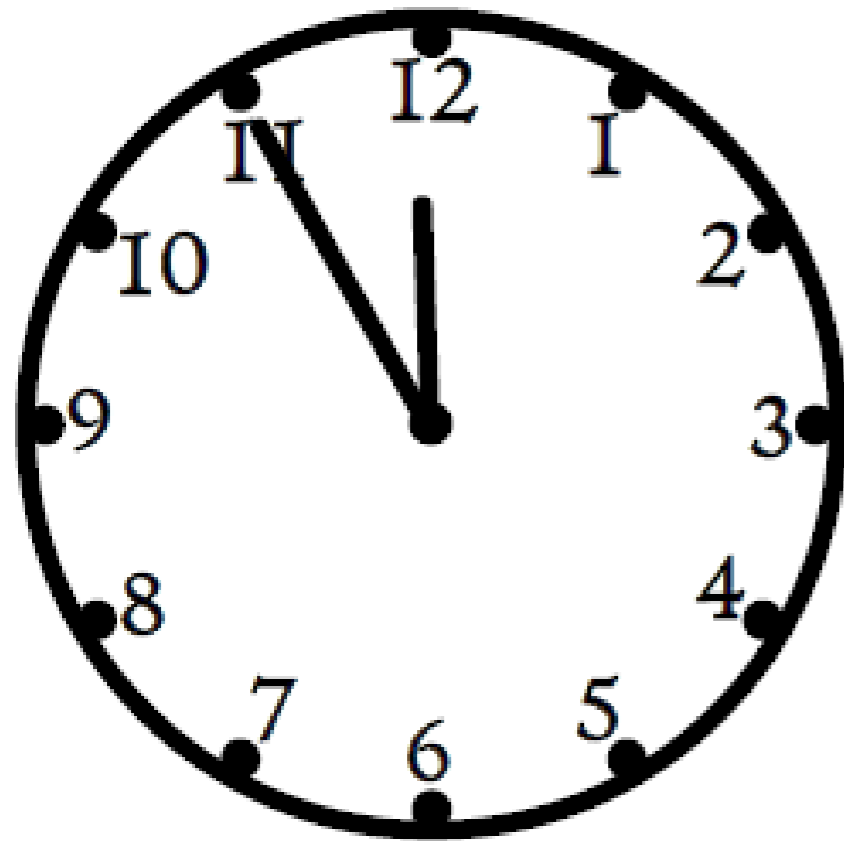
$7 + 3 + \square$

$6 + \square$

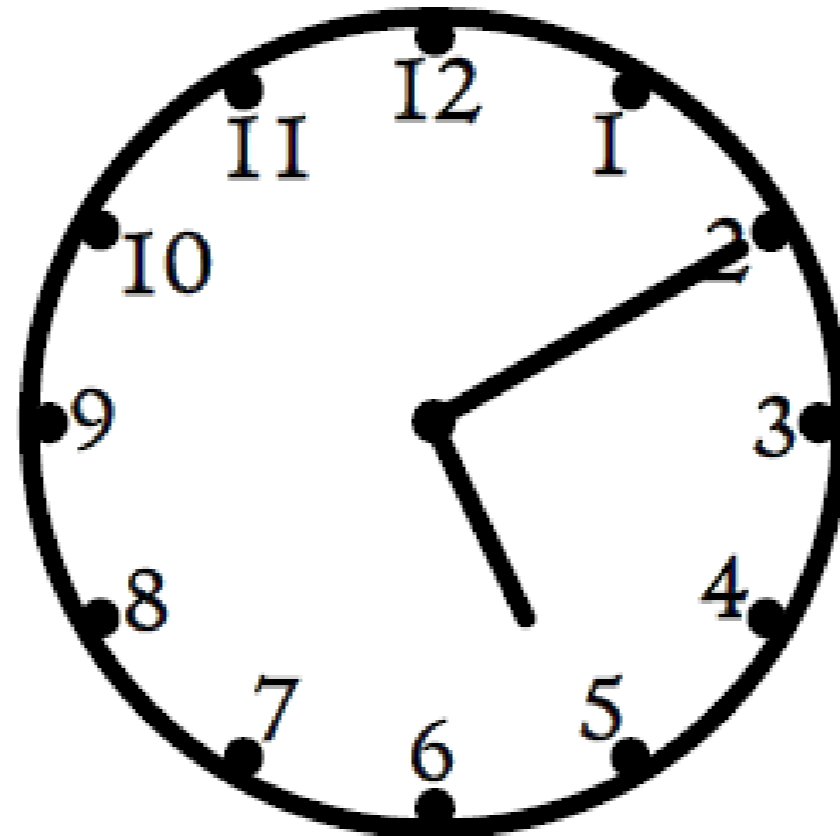
Spot the Difference



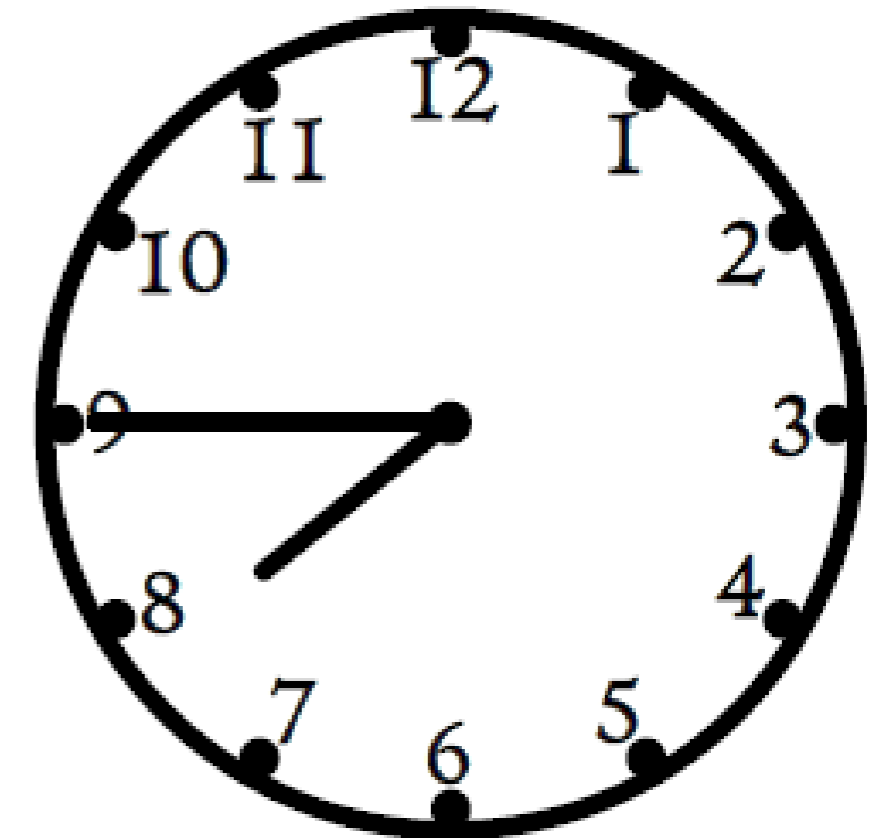
Explain the Mistakes



11:00



5:02

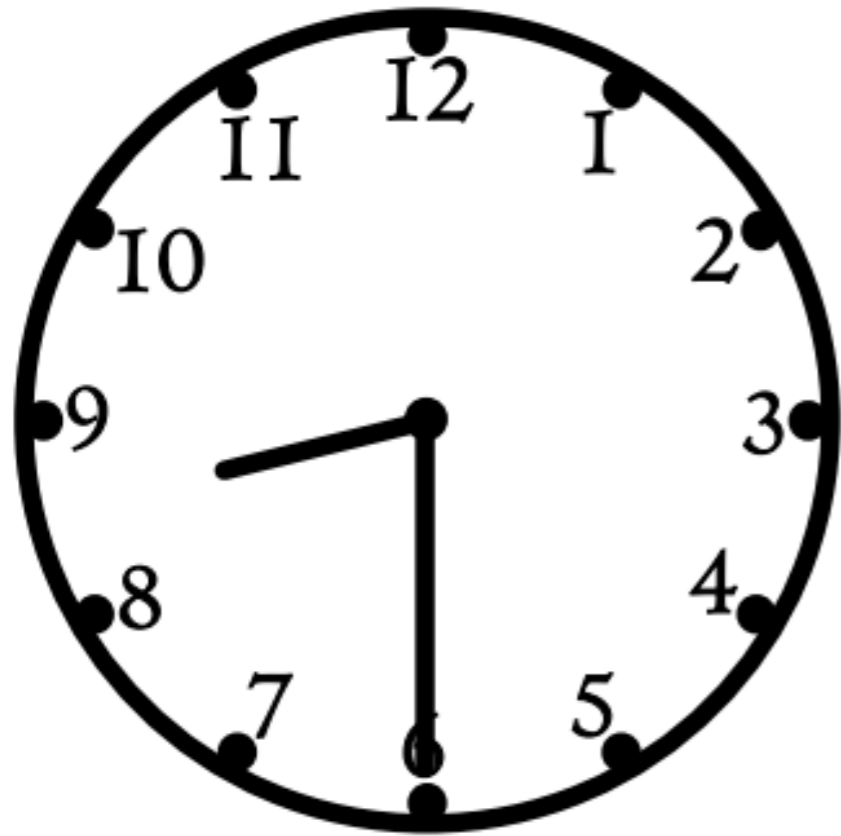


8:45

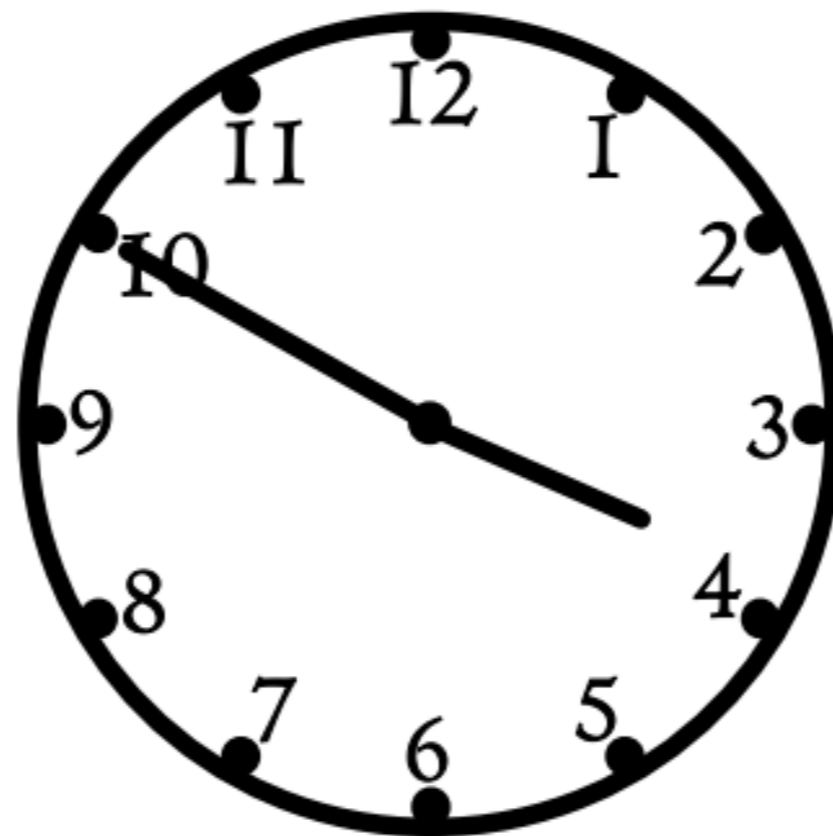
When the minute hand points to... it means...

If the time was... the hour hand would be...

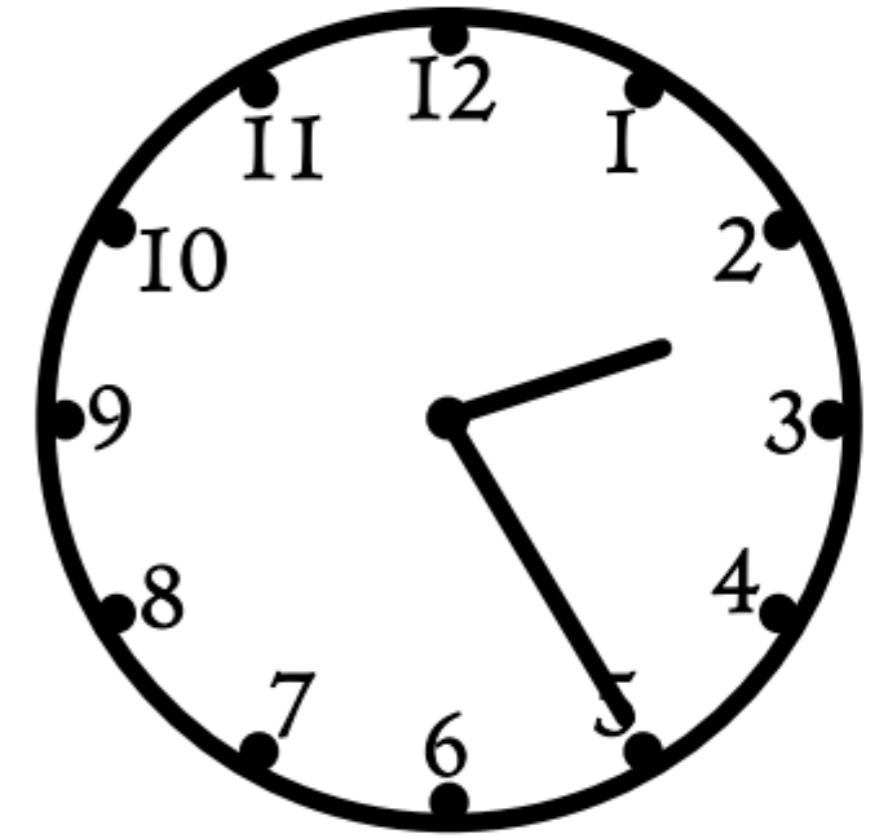
Which Answer?



8:30 **OR** 9:30



4:10 **OR** 3:50



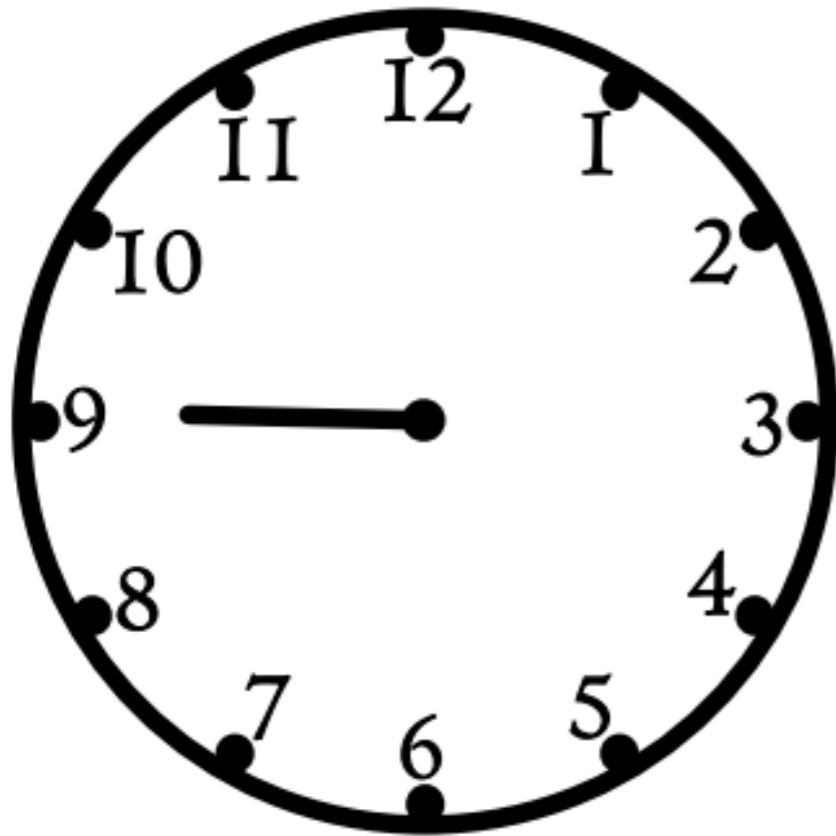
2:05 **OR** 2:25

Explain the mistakes.

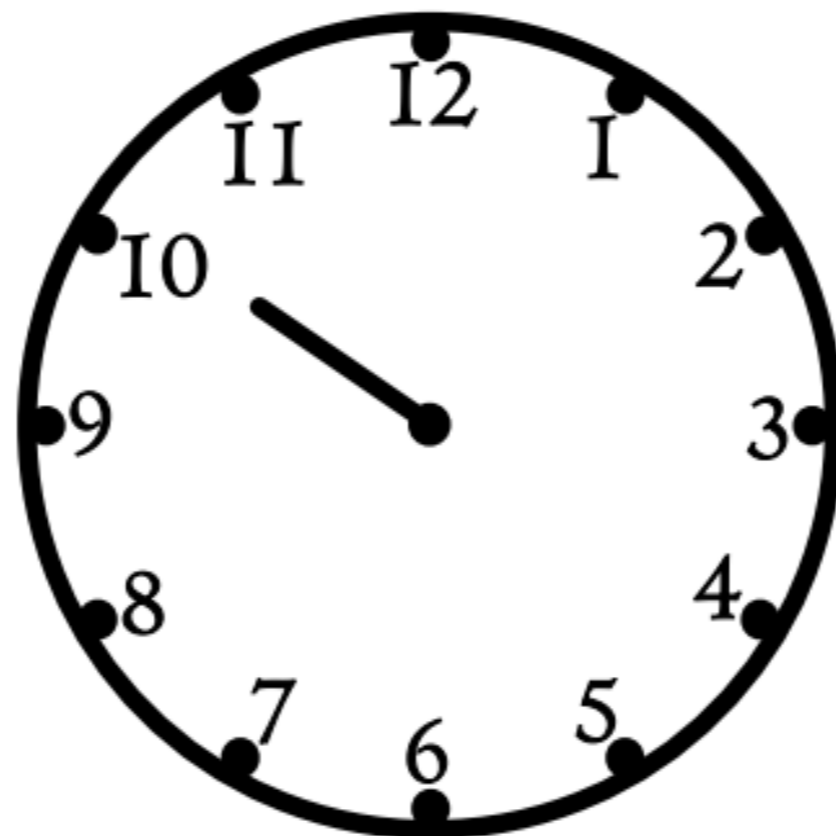
Read the Pictures

Draw the missing hands:

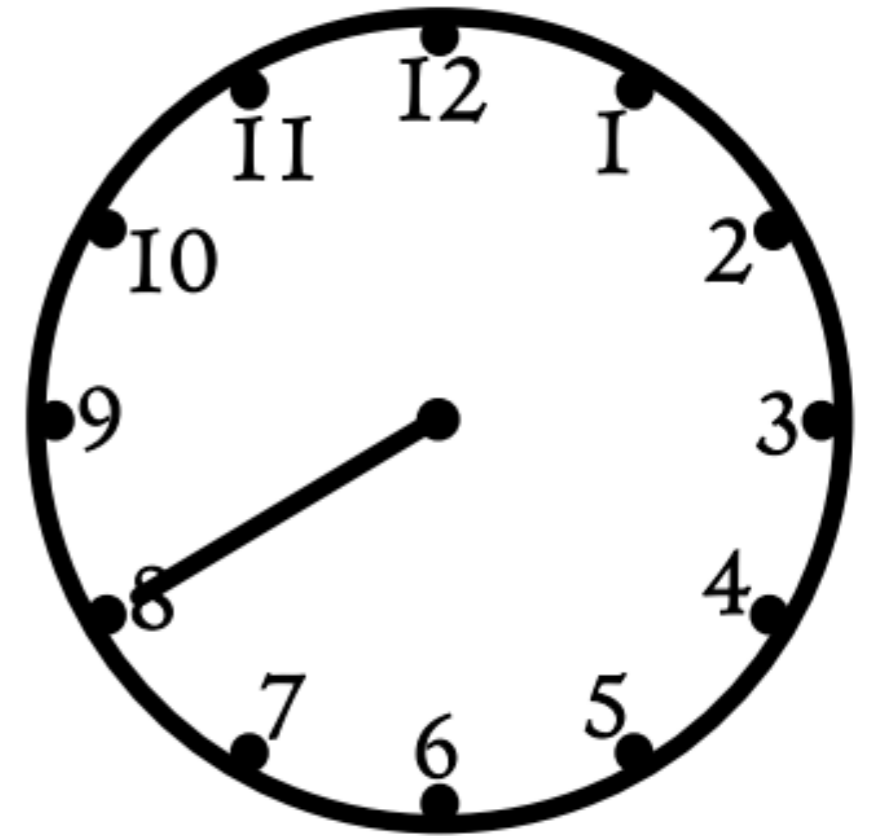
9:03



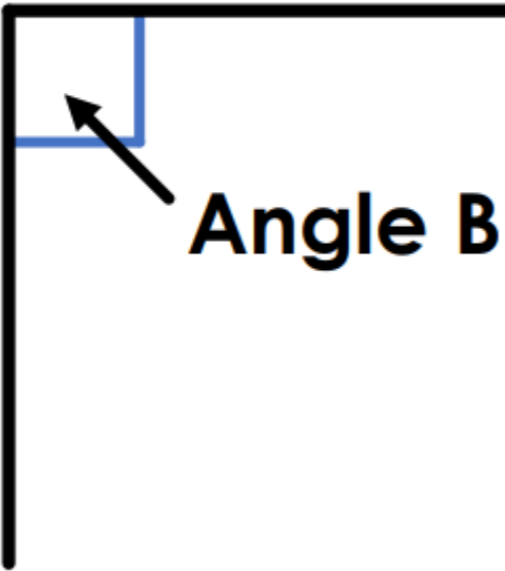
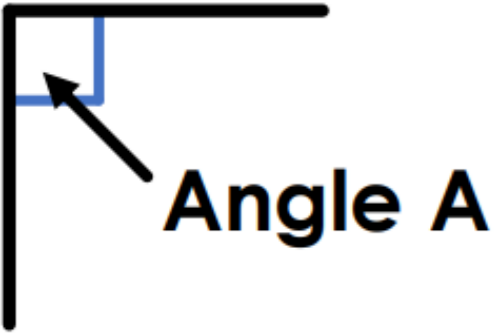
10:10



3:40



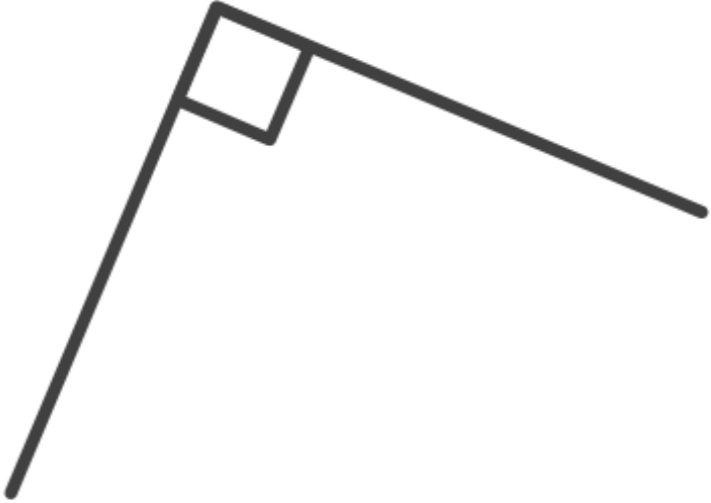
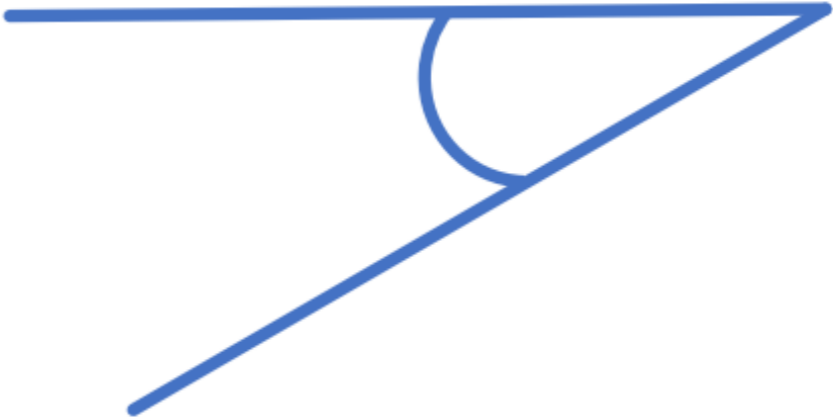
Explain the Mistake



Angle A is a smaller right angle than angle B

Read the Pictures

Order the angles from smallest to largest:



Small Difference Questions

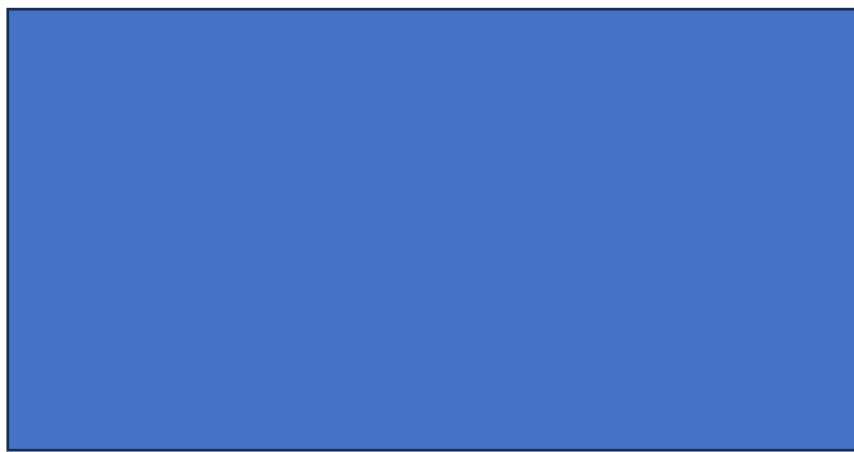
$$12 \div 3 =$$

$$13 \div 3 =$$

$$14 \div 3 =$$

$$15 \div 3 =$$

dividend \div divisor = quotient



Small Difference Questions

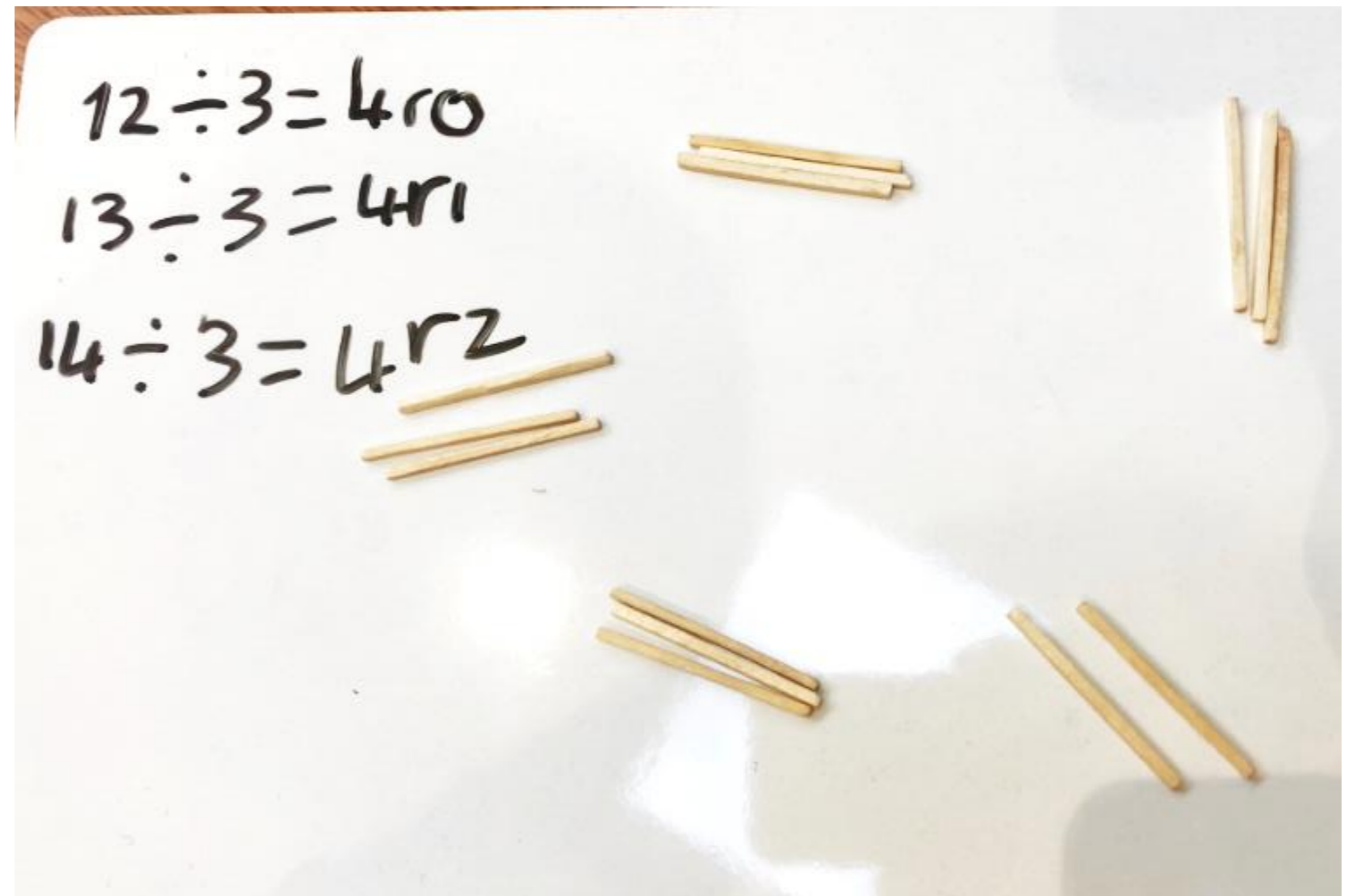
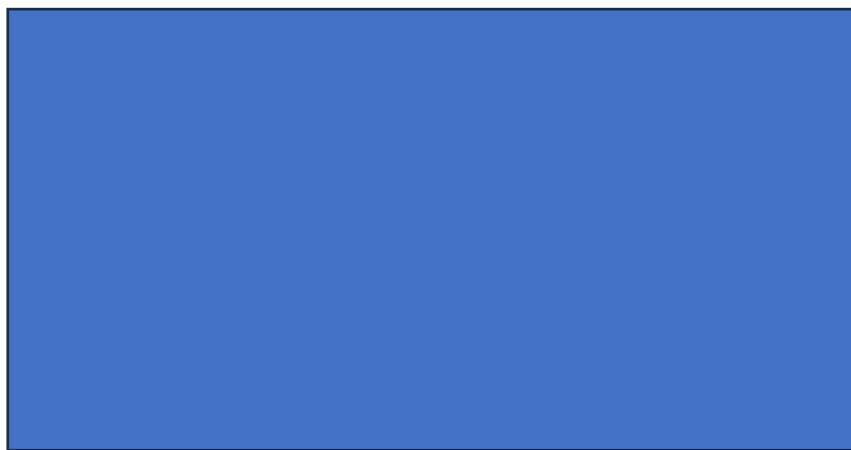
$$12 \div 3 = 4$$

$$13 \div 3 = 4 \text{ r } 1$$

$$14 \div 3 = 4 \text{ r } 2$$

$$15 \div 3 = 5$$

dividend \div divisor = quotient



Small Difference Questions

$$12 \div 3 = 4$$

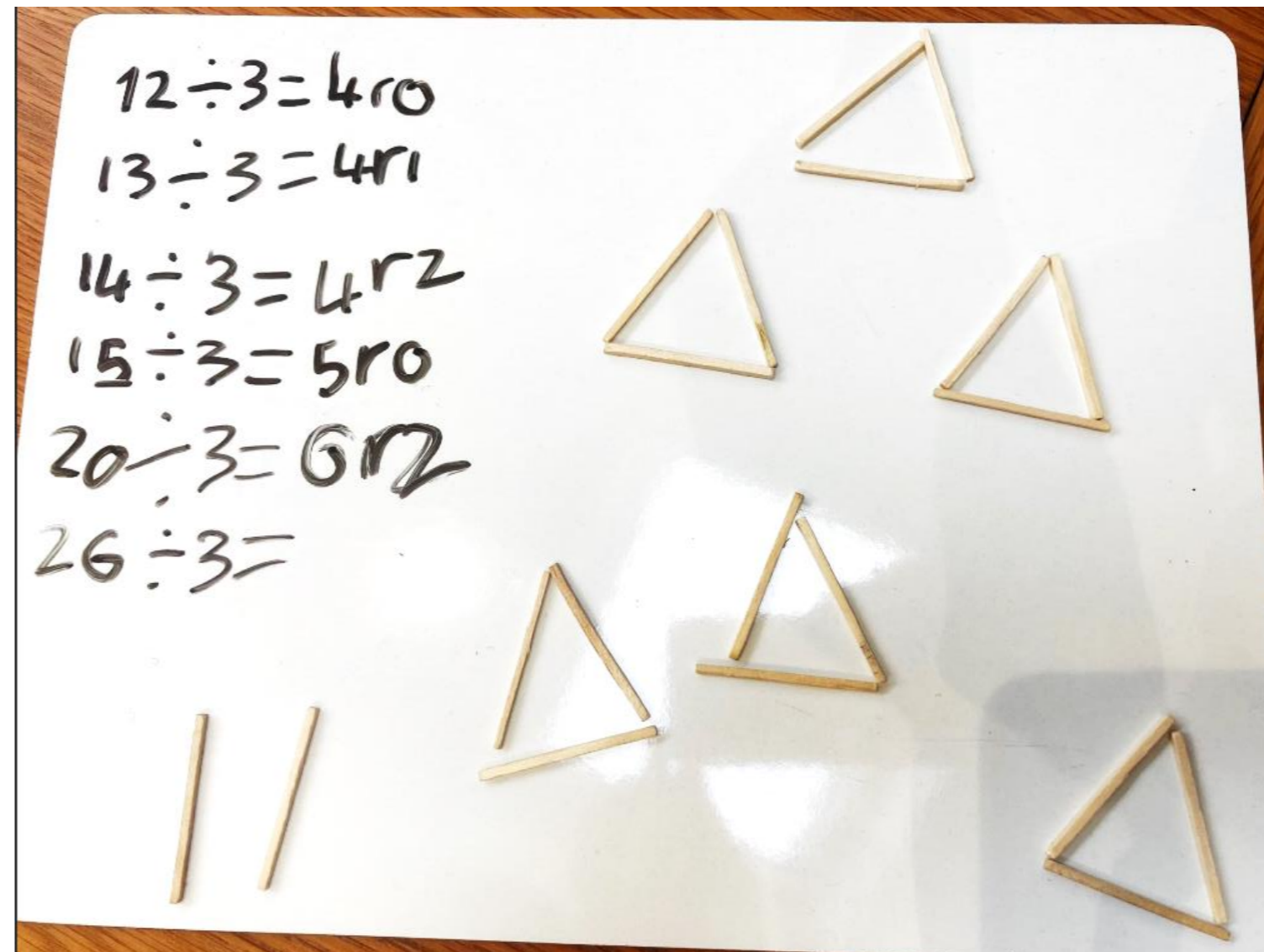
$$13 \div 3 = 4 \text{ r } 1$$

$$14 \div 3 = 4 \text{ r } 2$$

$$15 \div 3 = 5$$

$$20 \div 3 =$$

dividend \div divisor = quotient



Small Difference Questions

$$12 \div 3 = 4$$

$$13 \div 3 = 4 \text{ r } 1$$

$$14 \div 3 = 4 \text{ r } 2$$

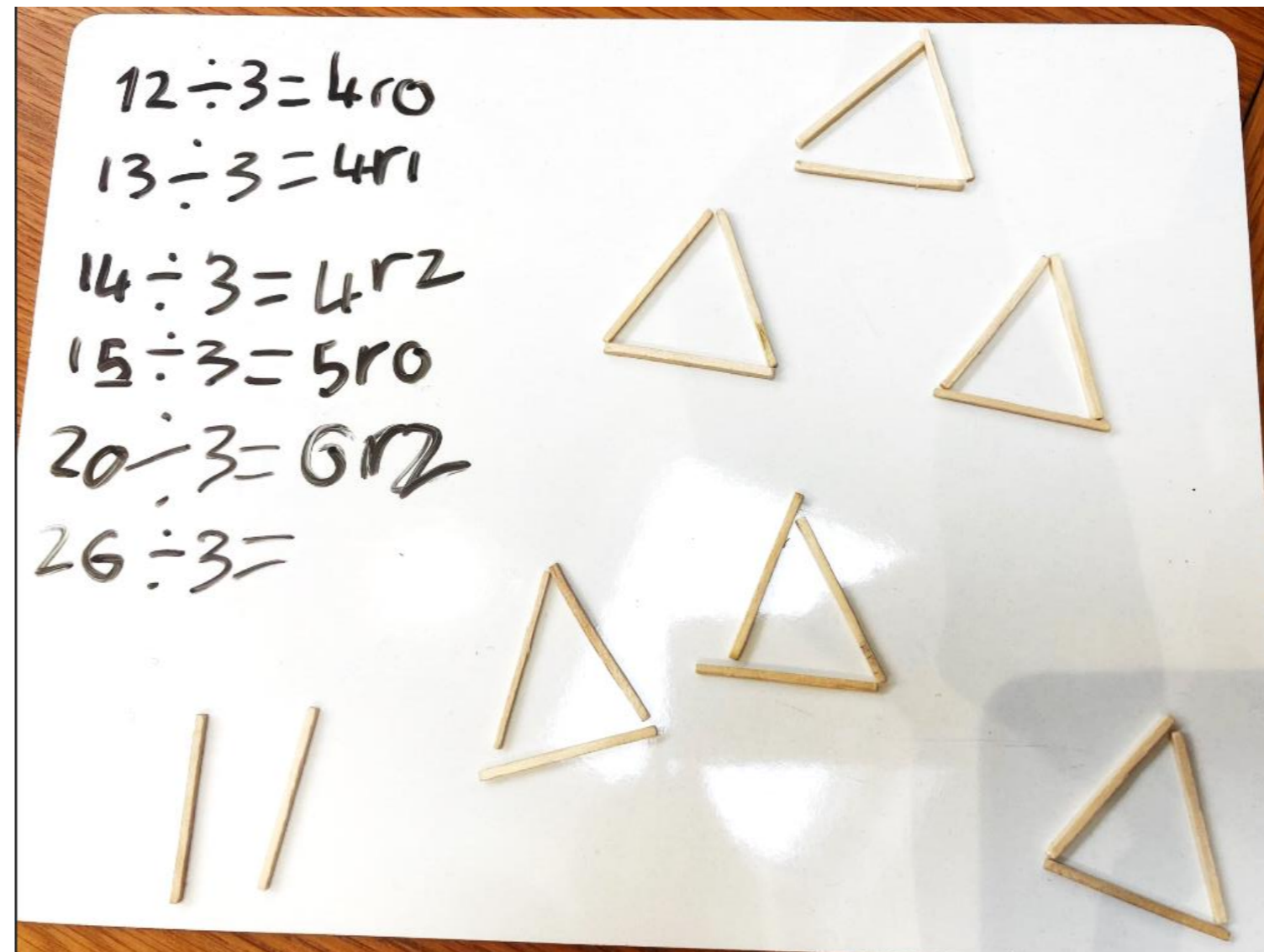
$$15 \div 3 = 5$$

$$20 \div 3 = 6 \text{ r } 2$$

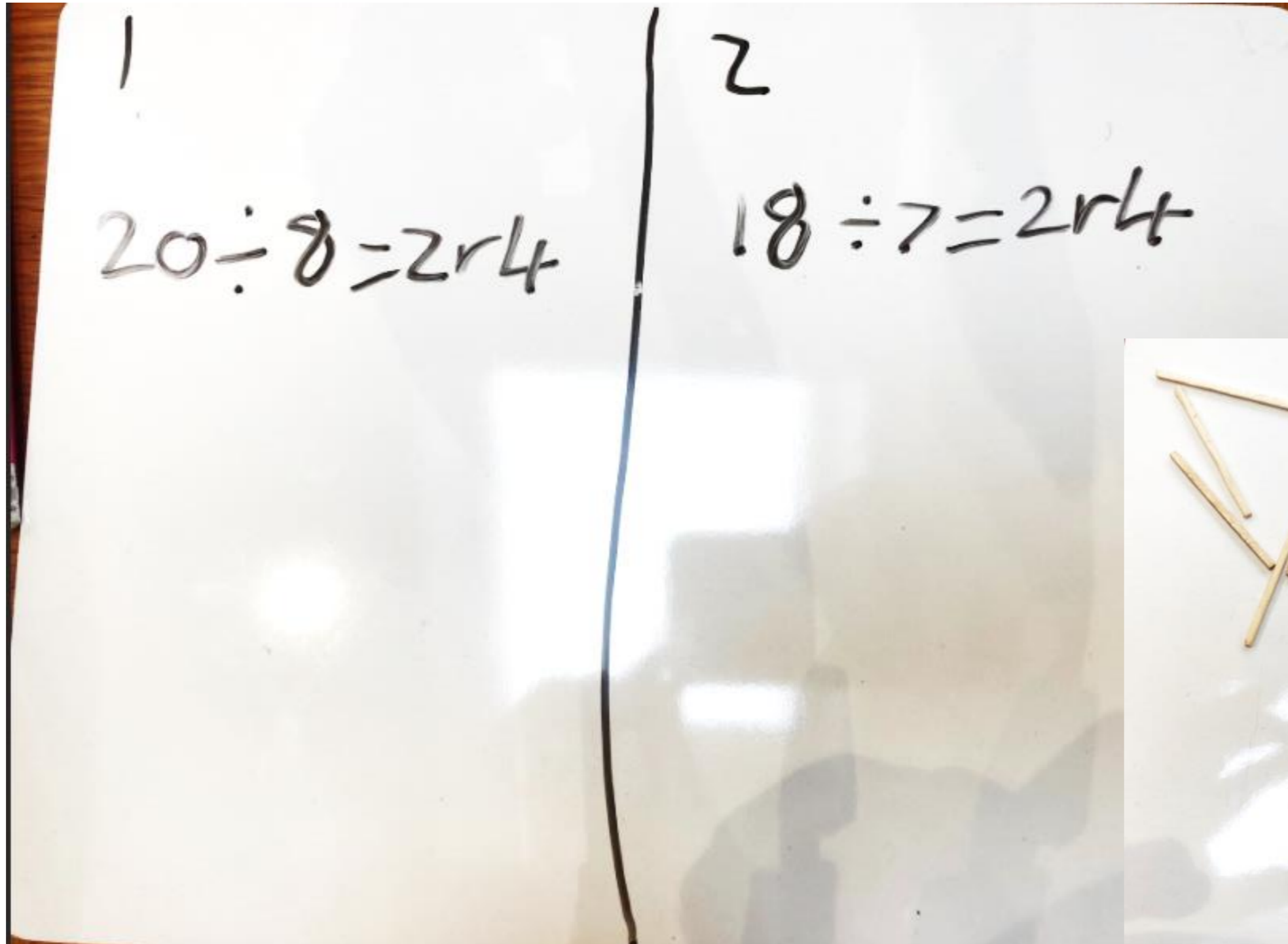
$$26 \div 3 =$$



dividend \div divisor = quotient



$$\square \div \square = 2 \text{ r } 4$$



Patterns in Division

I know... so...

$$20 \div 5 = 4$$

$$16 \div 4 = 4$$

$$18 \div 3 = 6$$

$$12 \div 3 = 4$$

$$24 \div 5 = 4r4$$

$$15 \div 4 = 3r3$$

$$21 \div 3 = 7r0$$

$$16 \div 3 = 5r1$$

Small Difference Questions

$$10 \div 4 = 2r2$$

$$11 \div 4 = 2r3$$

$$12 \div 4 = 3r0$$

$$12 \div 8 = 1r4$$

$$24 \div 8 = 3r0$$

Small Difference Questions

$$375 + 145 = \boxed{520}$$

$$345 + 175 = \boxed{520}$$

$$345 + 157 = \boxed{502}$$

$$353 + 149 = \boxed{502}$$

Small Difference Questions

253 rounded to the **nearest 100** is

300

253 rounded to the **nearest 10** is

250

248 rounded to the **nearest 10** is

250

248 rounded to the **nearest 100** is

200

Small Difference Questions

$$15 \times 3 = \boxed{45}$$

$$15 = 3 \times \boxed{5}$$

$$15 \div 3 = \boxed{5}$$

$$\boxed{5} = 15 \div 3$$

$$\boxed{45} \div 15 = 3$$

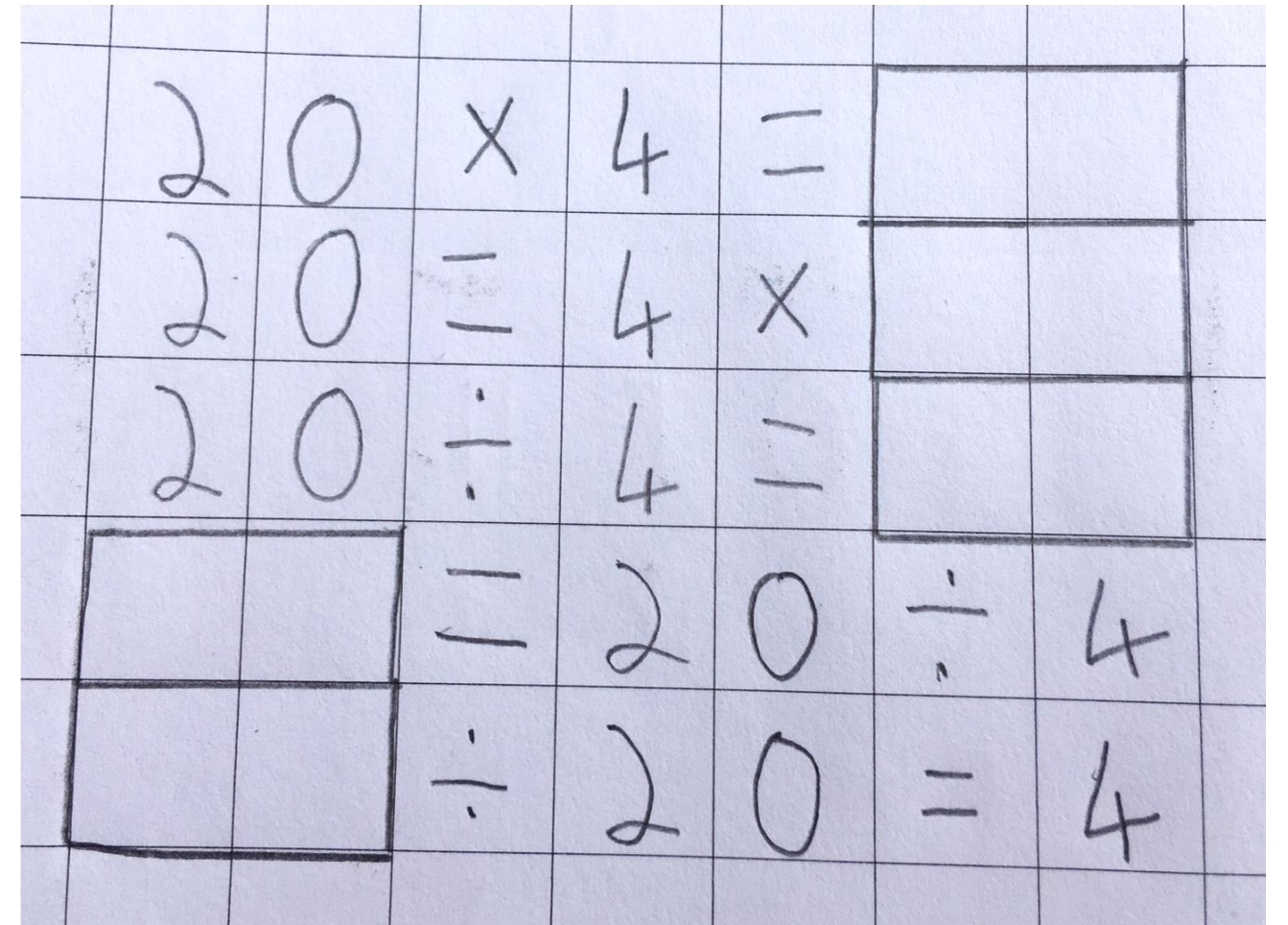
$$12 \div \boxed{3} = 4$$

$$12 \times 4 = \boxed{48}$$

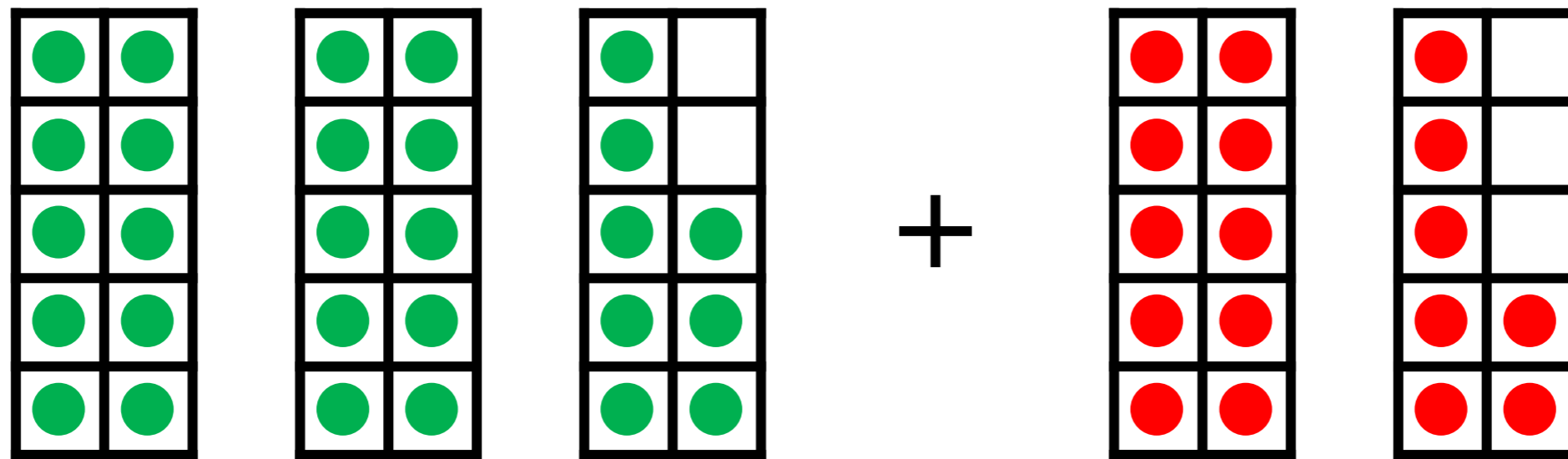
$$\boxed{48} \div 4 = 12$$

$$4 = 12 \div \boxed{3}$$

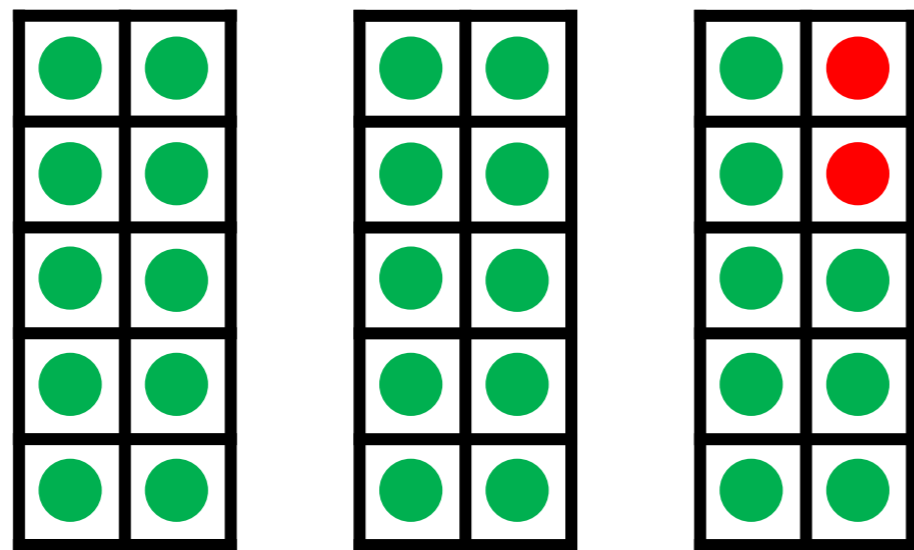
$$4 = \boxed{48} \div 12$$



$$28 + 17$$

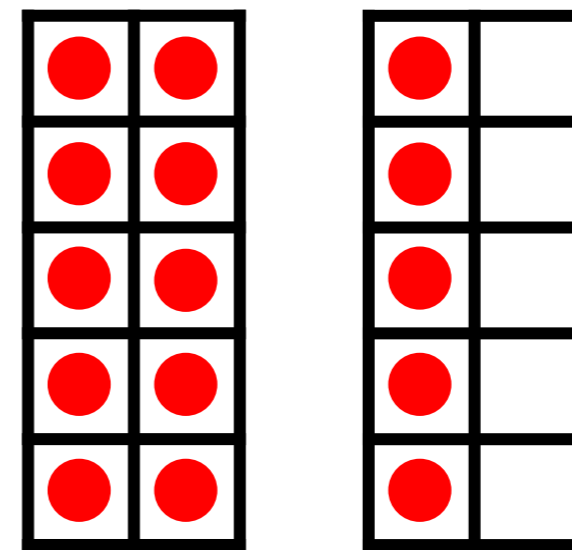


$28 + 17$

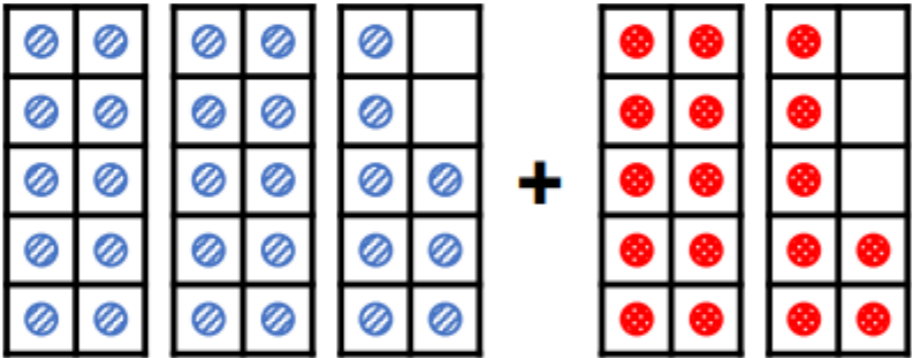


$30 + 15$

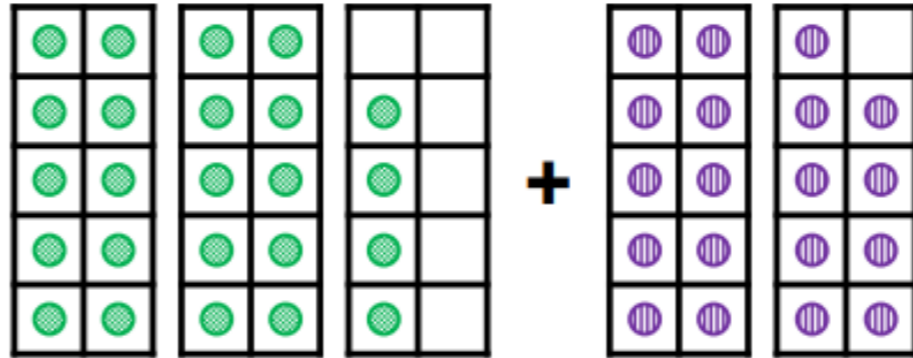
+



Read the Pictures

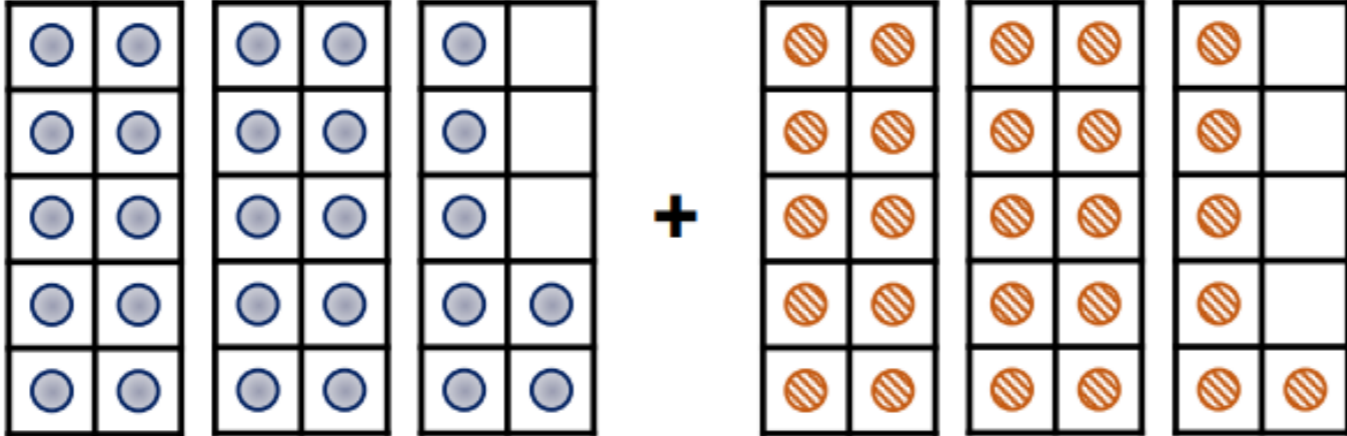


$28 + 17$ is equal to $30 + \square$
 $28 + 17 = \square$



$24 + 19$ is equal to $\square + 20$
 $24 + 19 = \square$

Different Methods



$27 + 26 = \square$

$30 + \square$

Double 20 add \square

Double \square add 3

Agree or Disagree?

✓ or ✗

55 + 35 is equal
to double 40

48 + 47 is equal
to 100 - 5

68 + 36 is equal to
70 + 38

54 + 28 is equal
to 52 + 30

Small Difference Questions

$28 + 6 = \square$

$23 + 56 = \square$

$40 + 35 = \square$

$26 + 8 = \square$

$53 + 26 = \square$

$39 + 35 = \square$

$36 + 18 = \square$

$53 + 28 = \square$

$38 + 34 = \square$

$34 + 20 = \square$

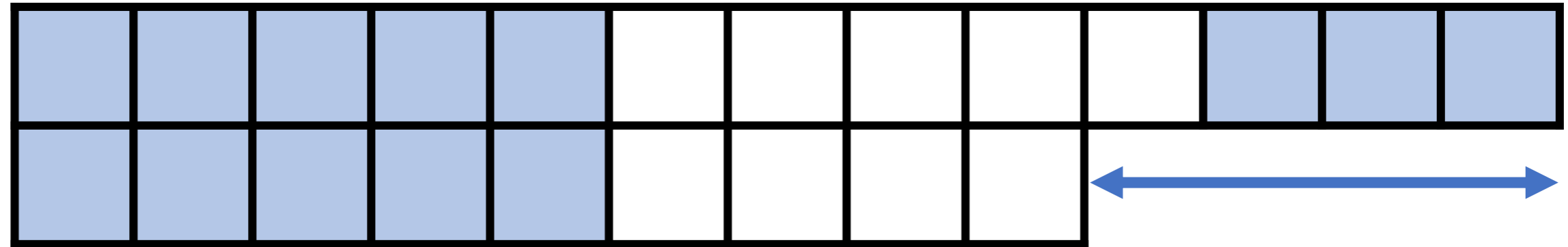
$51 + 30 = \square$

$58 + 14 = \square$

The answer to... and... is the same because...

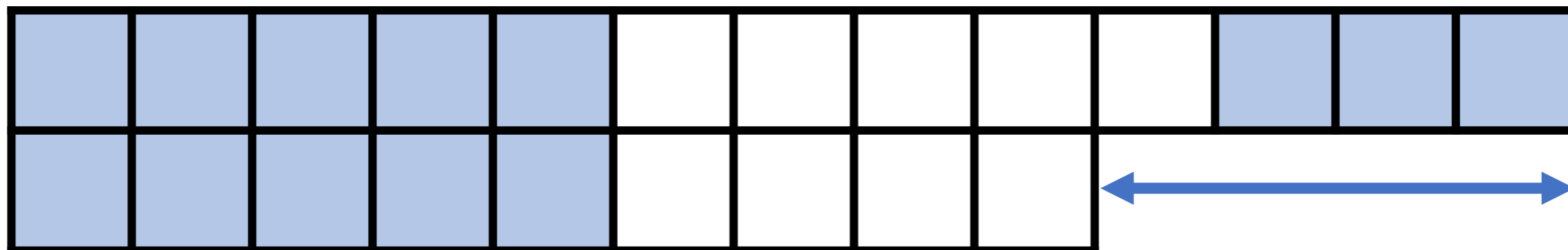
I used the answer to... to help me to calculate...

$$13 - 9 = 4$$

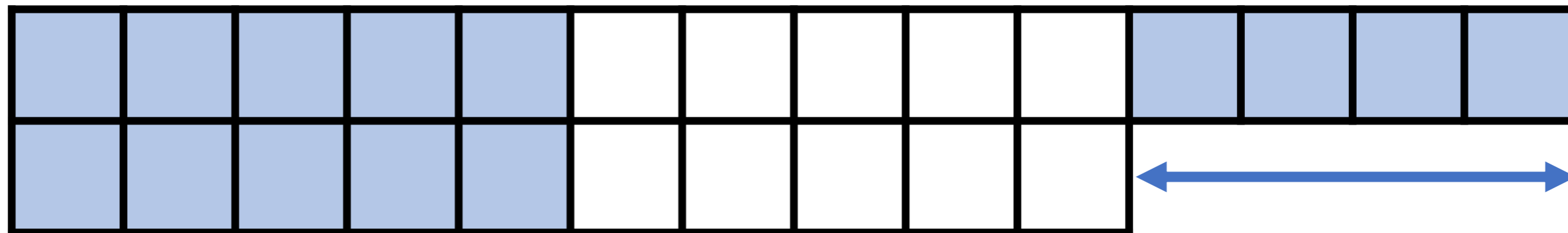




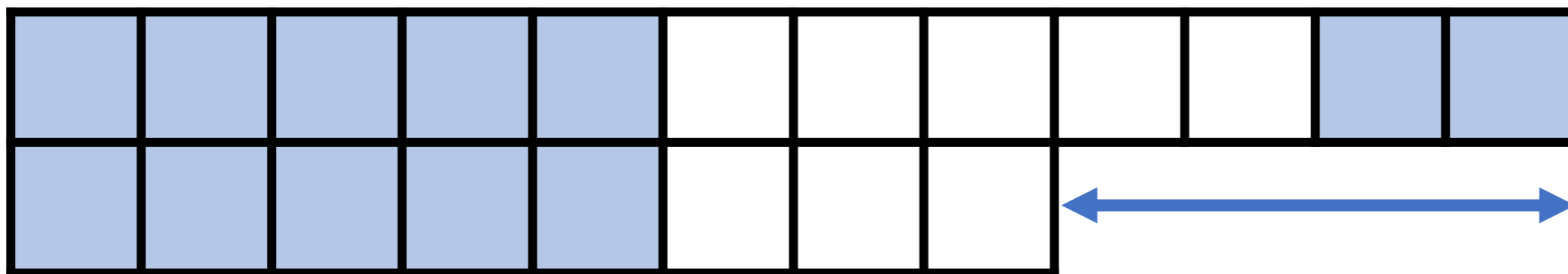
$$13 - 9 = 4$$



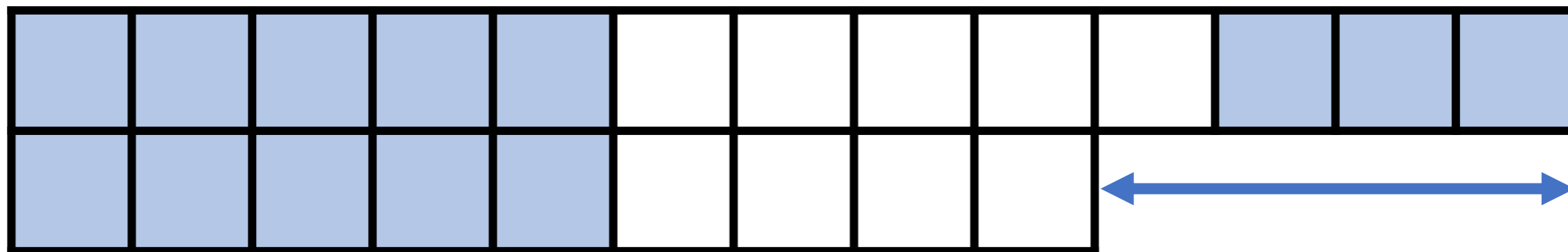
$$14 - 10 = 4$$



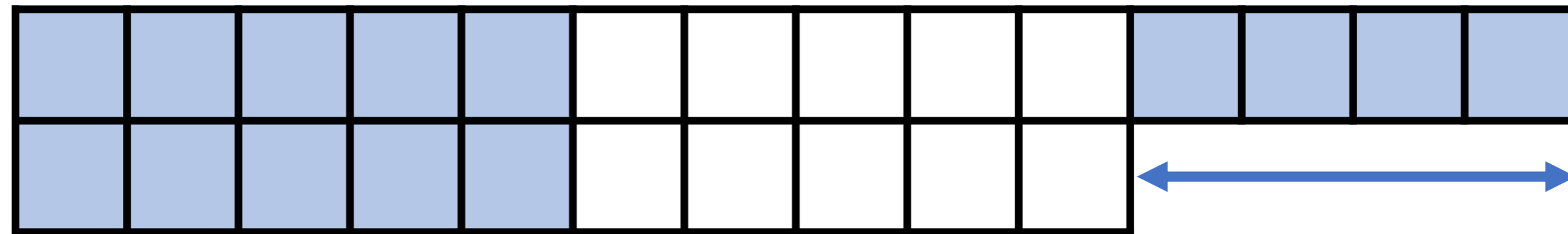
$$12 - 8 = 4$$

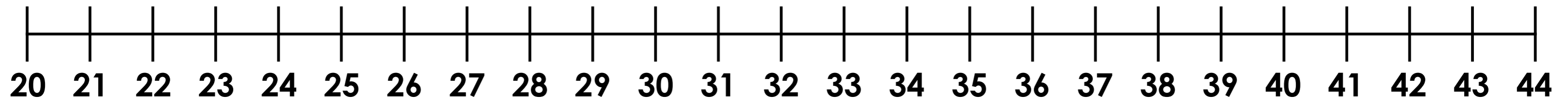


$$13 - 9 = 4$$

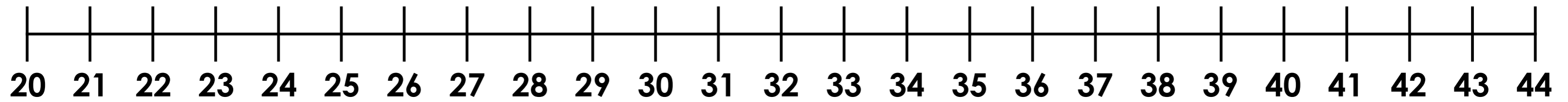


$$14 - 10 = 4$$





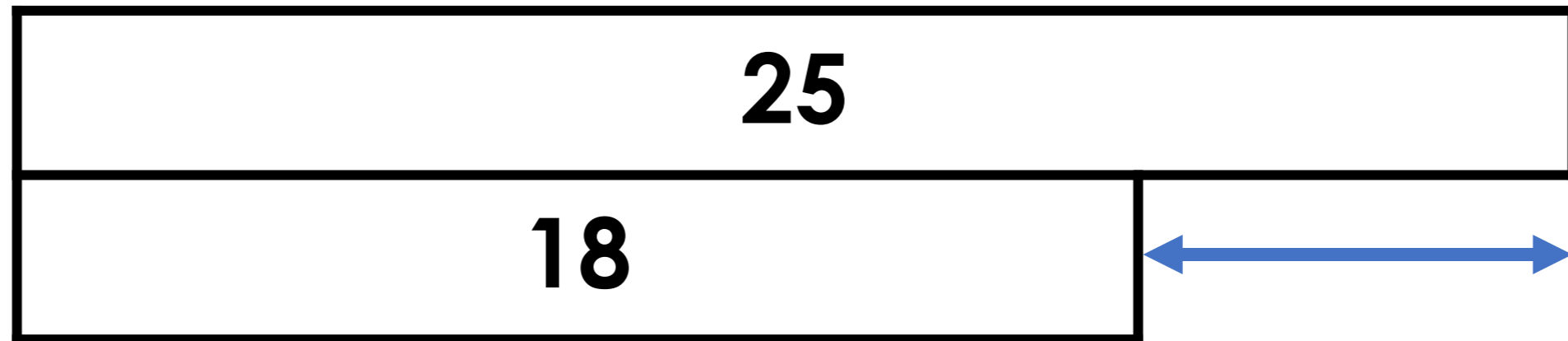
$$41 - 28$$



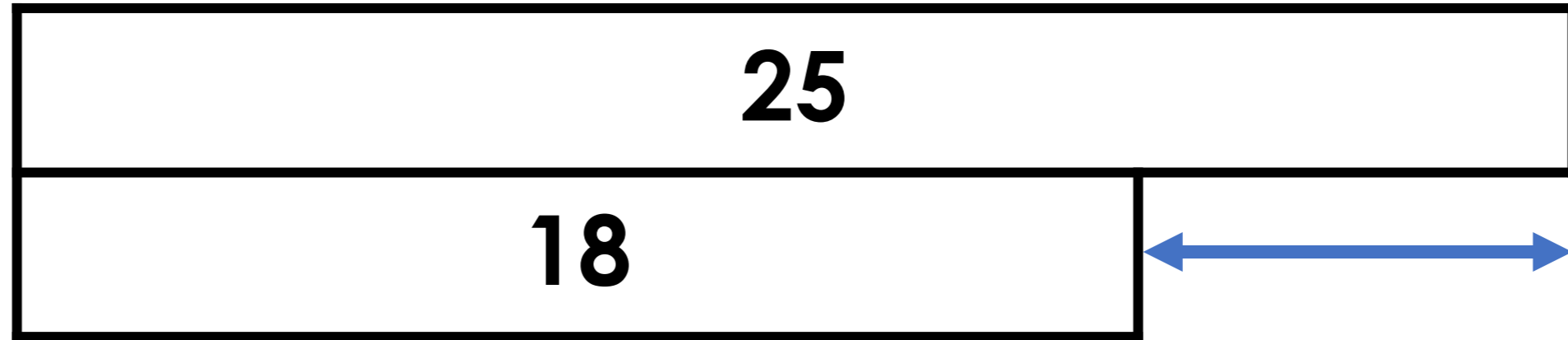
$$41 - 28$$

$$43 - 30$$

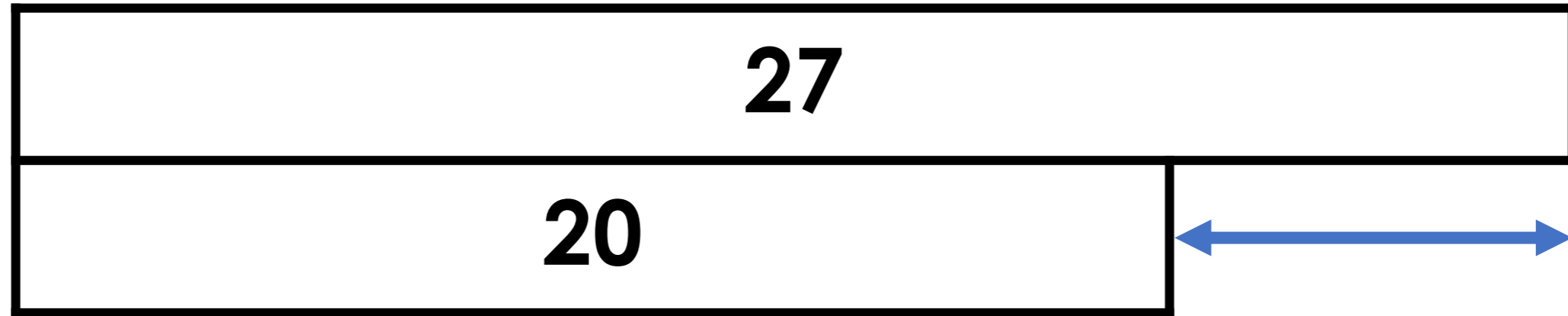
$25 - 18 =$



$25 - 18 =$



$27 - 20 =$



Rank by Difficulty

$$601 - 337 =$$

$$869 - 321 =$$

$$845 - 297 =$$

Maths

$$\begin{array}{r} 601 \\ - 337 \\ \hline 264 \end{array}$$

Hardest
because
you had
to carry,
lots.

$$\begin{array}{r} 869 \\ - 321 \\ \hline 548 \end{array}$$

~~Easy~~
Easiest
you don't have to carry.

$$\begin{array}{r} 845 \\ - 297 \\ \hline 548 \end{array}$$

middle.

Subtraction Bordering Tens

Task E

Small Difference Questions

$7 - 5 = \boxed{2}$

$17 - 5 = \boxed{12}$

$15 - 7 = \boxed{8}$

$14 - 6 = \boxed{8}$

$34 - 6 = \boxed{28}$

$44 - 16 = \boxed{28}$

$9 - 3 = \boxed{6}$

$9 - \boxed{6} = 3$

$\boxed{12} - 9 = 3$

$\boxed{14} - 9 = 5$

$\boxed{24} - 9 = 15$

$\boxed{24} - 15 = 9$

How does the question change?

How does the answer change?

$$(d) 64 + 83$$

$$(e) 71 + 76$$

$$(d) 24 \times 6$$

$$(e) 48 \times 3$$

$$(d) 78 - 56$$

$$(e) 81 - 59$$

$$(d) 24 \div 4$$

$$(e) 48 \div 8$$

$$35 + 24 = \underline{\quad} - 10$$

Correct or Incorrect?



blue = green + red + yellow

✓ or ✗

Correct or Incorrect?



✓ or ✗

blue = green + red + yellow

blue - red = green + yellow

Correct or Incorrect?



✓ or ✗

blue = green + red + yellow

blue - red = green + yellow

blue + yellow = red + green

Correct or Incorrect?



✓ or ✗

blue = green + red + yellow

blue - red = green + yellow

blue + yellow = red + green

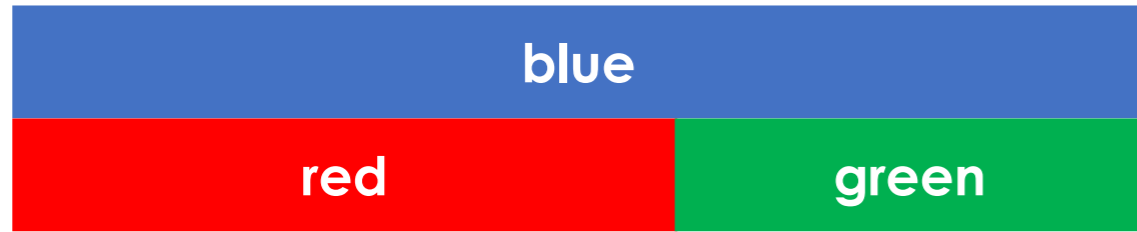
blue - green = red + yellow

Question A:

$$35 + 24 = \underline{\quad}$$

Question B:

$$35 + 24 = \underline{\quad} - 10$$

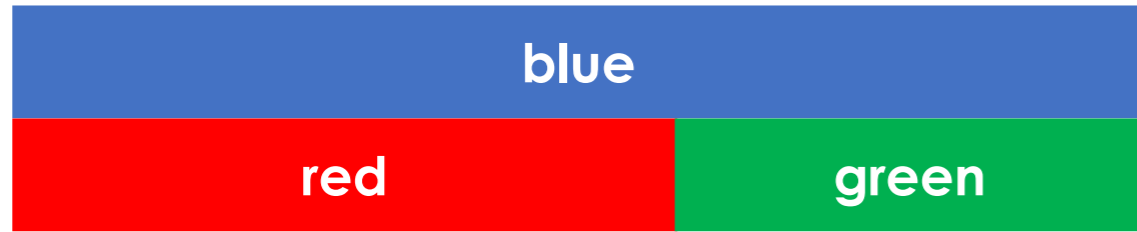


Question A:

$$35 + 24 = \underline{\quad}$$

Question B:

$$35 + 24 = \underline{\quad} - 10$$



Question A:

$$35 + 24 = \underline{\quad}$$



Question B:

$$35 + 24 = \underline{\quad} - 10$$

Balancing Equations

$$42 + 16 = \underline{\quad} + 10$$

$$40 - 7 = 25 + \underline{\quad}$$

Representation(s)

Pairs of examples

Which Answer?

Small Difference Questions

Which Answer?

$$42 + 16 = \underline{\quad} + 10$$

48

58

68

Small Difference Questions

$$\square + 15 = 40$$

$$\square + 15 = 40 + 10$$

$$\square + 15 = 40 - 10$$

$$\square + 15 = 40 - 15$$

$$\square - 15 = 40 - 15$$

Small Difference Questions

$$\boxed{25} + 15 = 40$$

$$\boxed{} + 15 = 40 + 10$$

$$\boxed{} + 15 = 40 - 10$$

$$\boxed{} + 15 = 40 - 15$$

$$\boxed{} - 15 = 40 - 15$$

Small Difference Questions

$$\boxed{25} + 15 = 40$$

$$\boxed{35} + 15 = 40 + 10$$

$$\boxed{} + 15 = 40 - 10$$

$$\boxed{} + 15 = 40 - 15$$

$$\boxed{} - 15 = 40 - 15$$

Small Difference Questions

$$\boxed{25} + 15 = 40$$

$$\boxed{35} + 15 = 40 + 10$$

$$\boxed{15} + 15 = 40 - 10$$

$$\boxed{} + 15 = 40 - 15$$

$$\boxed{} - 15 = 40 - 15$$

Small Difference Questions

$$\boxed{25} + 15 = 40$$

$$\boxed{35} + 15 = 40 + 10$$

$$\boxed{15} + 15 = 40 - 10$$

$$\boxed{10} + 15 = 40 - 15$$

$$\boxed{} - 15 = 40 - 15$$

Small Difference Questions

25

$$+ 15 = 40$$

35

$$+ 15 = 40 + 10$$

15

$$+ 15 = 40 - 10$$

10

$$+ 15 = 40 - 15$$

40

$$- 15 = 40 - 15$$