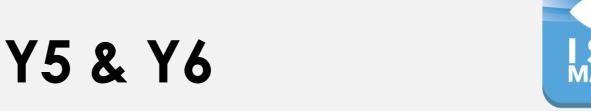
# Creating Reasoning Routines, Building Problem-Solvers

Session 2

Reasoning throughout knowledge acquisition

www.iseemaths.com



### Routines Within Interactive Teaching

#### Raising the internal narrative:

Gap between question and response/discussion, silent modelling

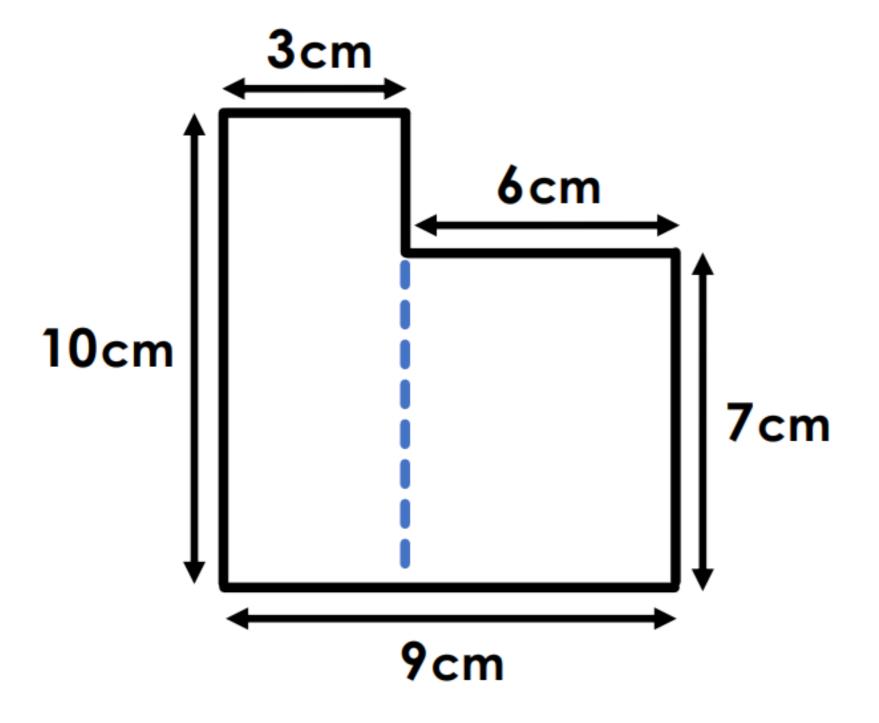
#### Mass participation:

Form of answer before question, routines

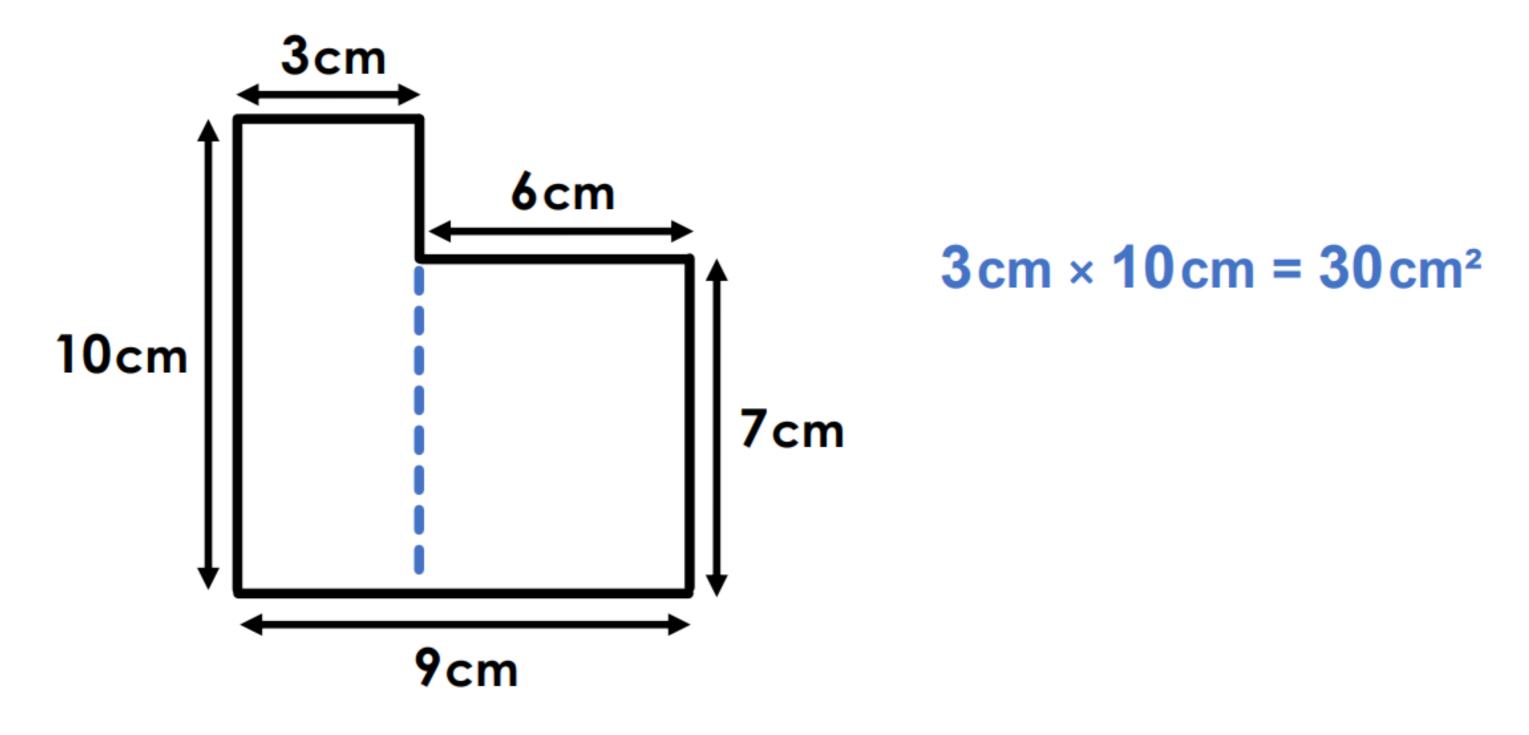
#### Managing discussions:

Quality modelling + participation, wait time 2

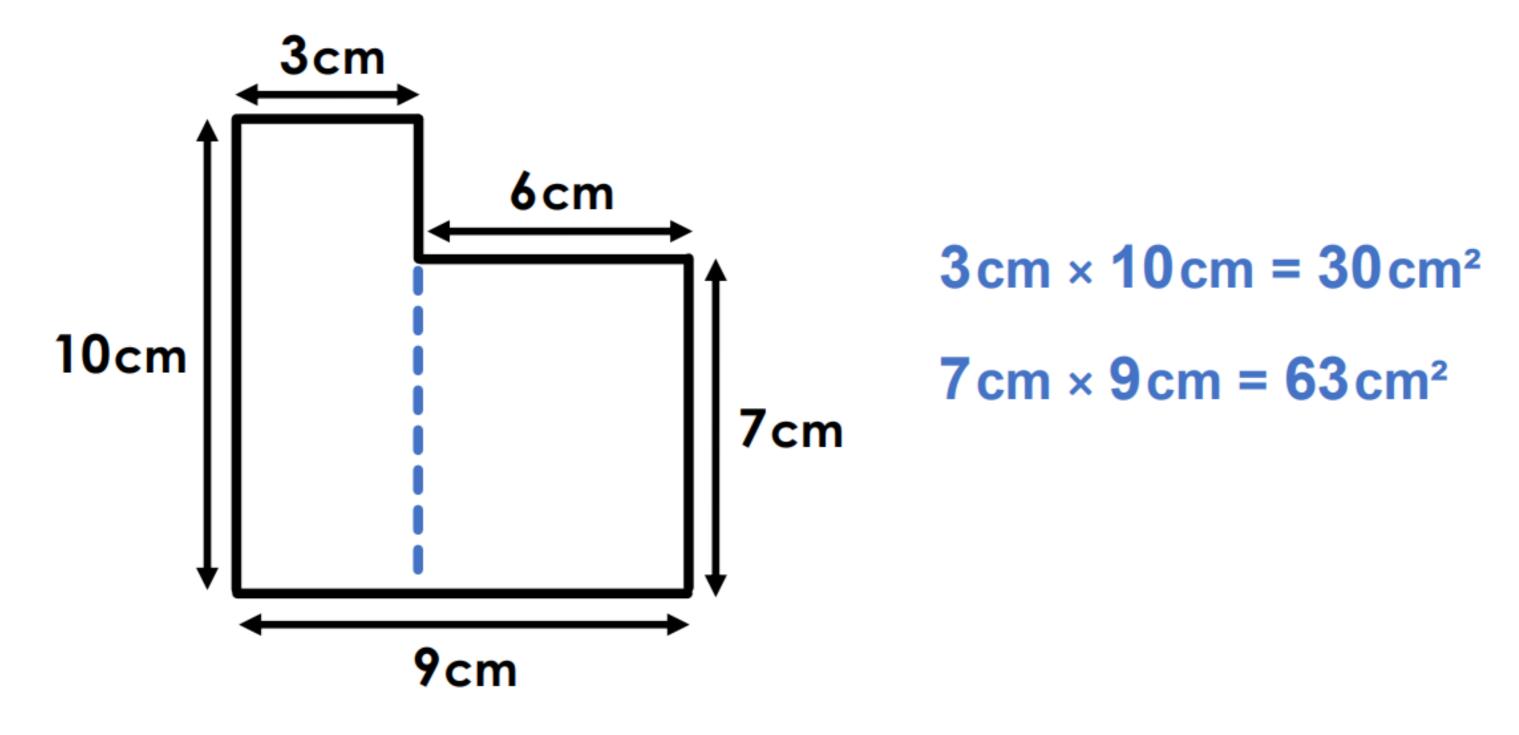
### Calculate the area of the shape:



### Calculate the area of the shape:



### Calculate the area of the shape:



There are 20 children at the park.

There are 4 times as many girls as boys at the park.

How many boys at the park?

There are 20 children at the park.

There are 4 times as many girls as boys at the park.

### How many boys at the park?



 $\frac{1}{4}$  of a number is 20.

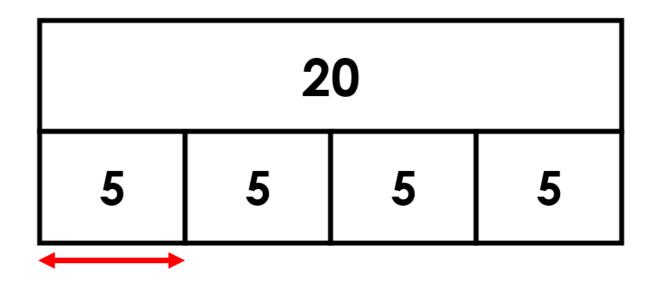
What is the number?

 $\frac{1}{4}$  of a number is 20.

## What is the number?



| 80 |    |    |    |  |
|----|----|----|----|--|
| 20 | 20 | 20 | 20 |  |

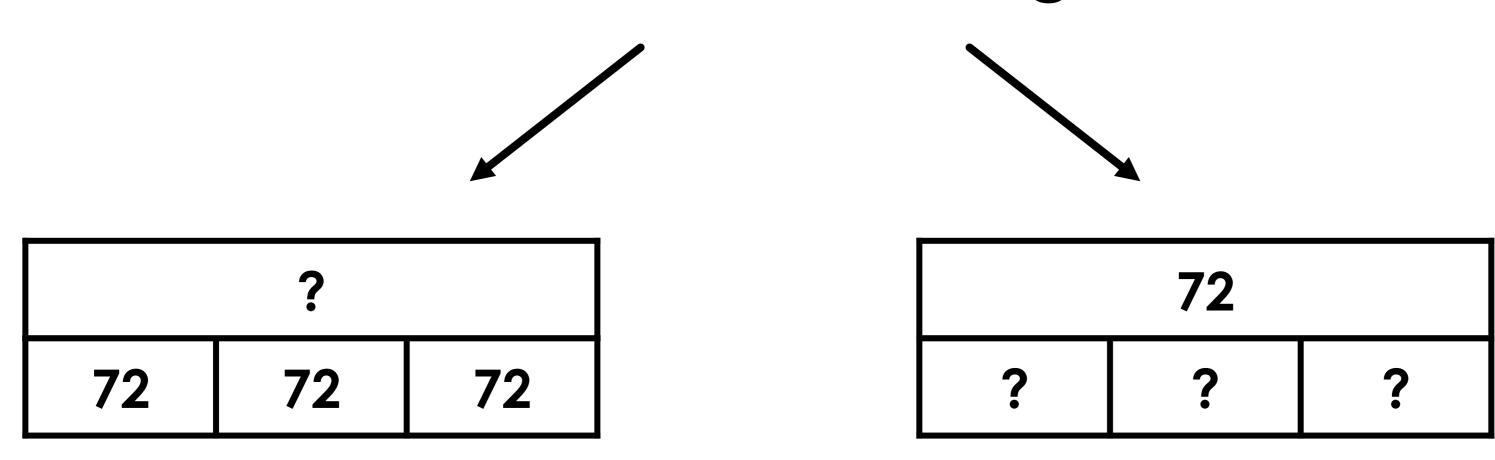


Ken thinks of a number. He divides it by 3. The answer is 72.

What number was Ken thinking of?

Ken thinks of a number. He divides it by 3. The answer is 72.

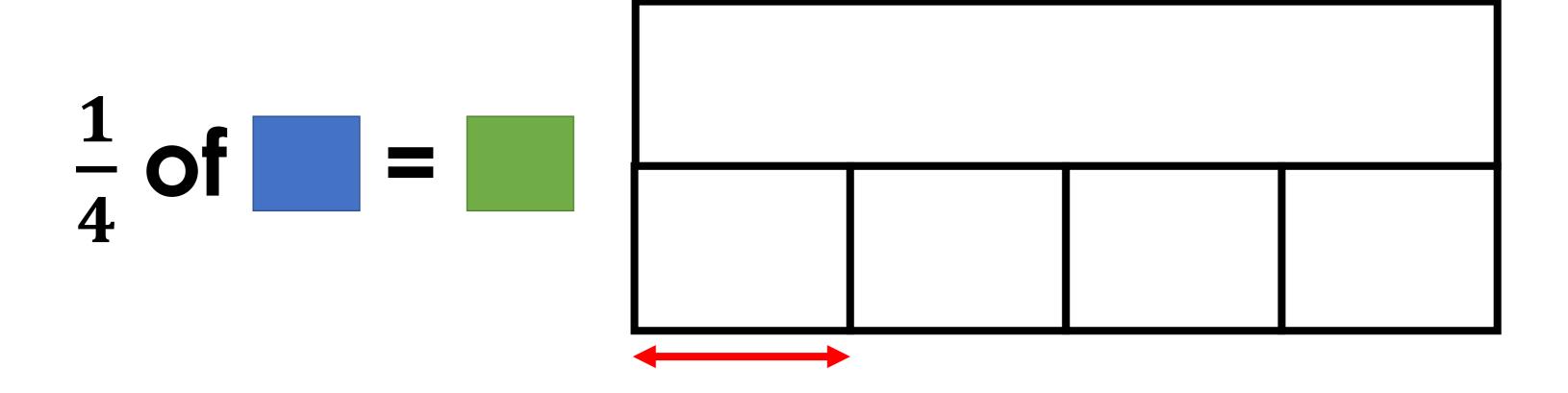
# What number was Ken thinking of?



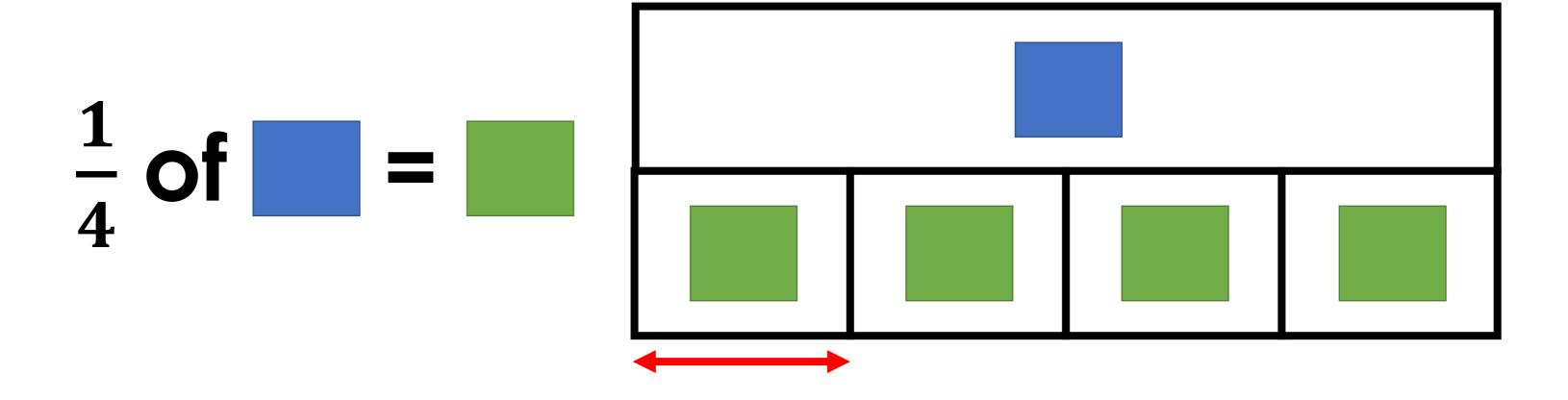


$$\frac{1}{4} \text{ of } = \boxed{\phantom{a}}$$



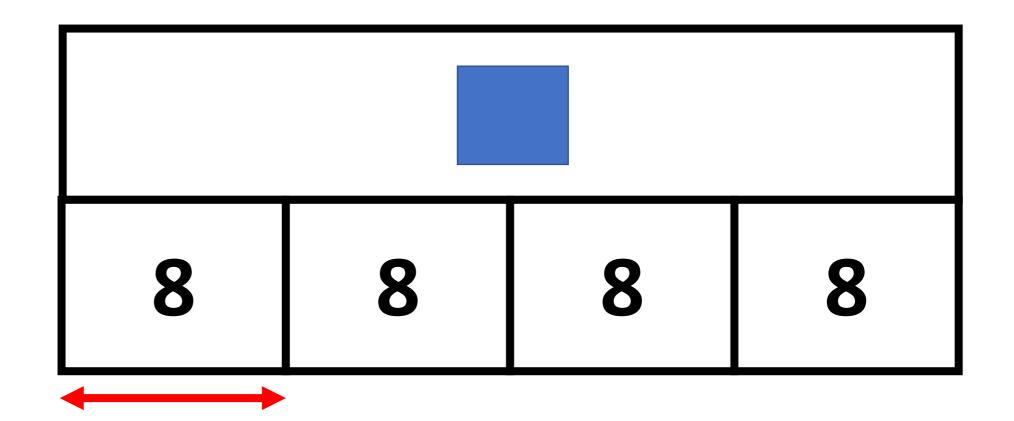






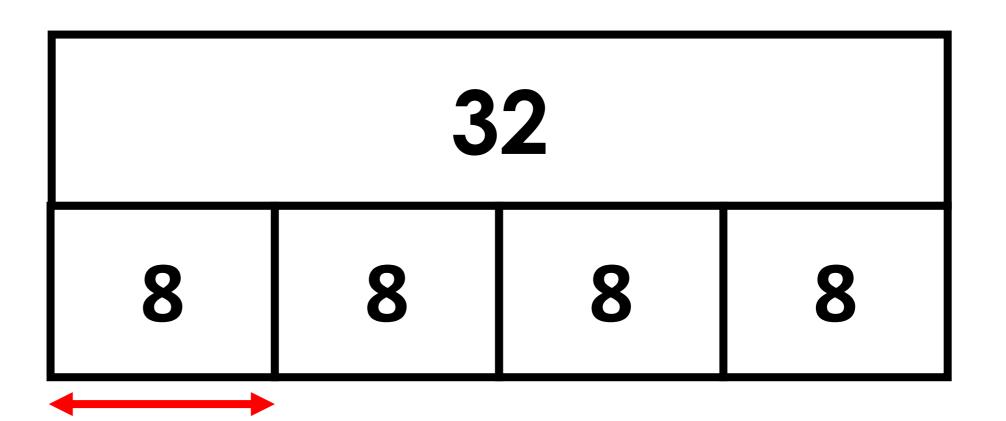


$$\frac{1}{4} \text{ of } = 8$$





$$\frac{1}{4}$$
 of 32 = 8



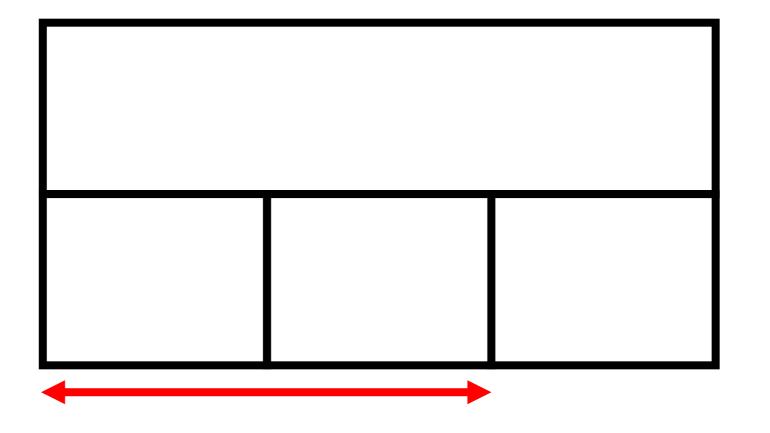
MAKE 32



$$\frac{2}{3} \text{ of } = \boxed{\phantom{a}}$$

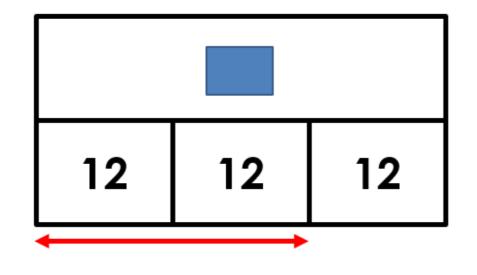


$$\frac{2}{3} \text{ of } = \boxed{\phantom{a}}$$

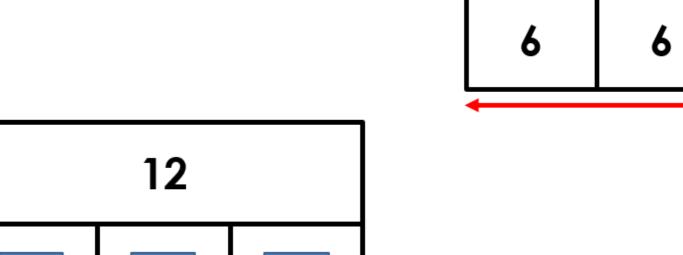


MAKE 32









6 6 6



A circus is holding a concert for charity.

Adult tickets cost £11. Child tickets cost £6.

How many child tickets are sold?

What information must be given?



A circus is holding a concert for charity.

Adult tickets cost £11. Child tickets cost £6.

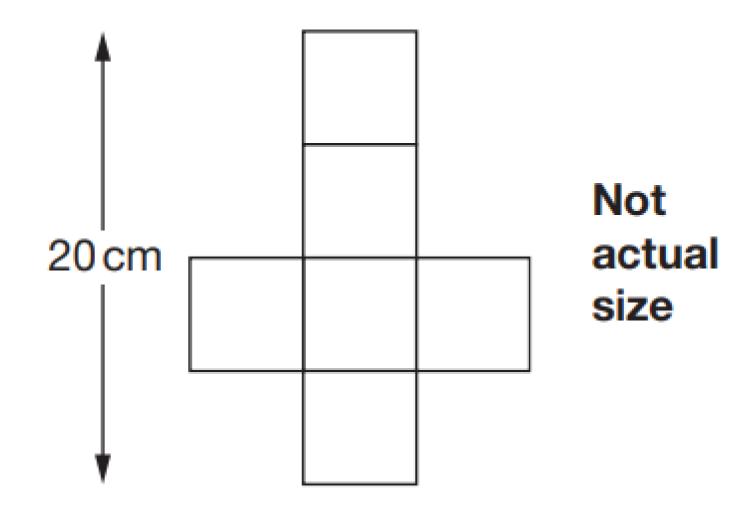
120 adult tickets are sold. In total, £1800 is raised.

How many child tickets are sold?

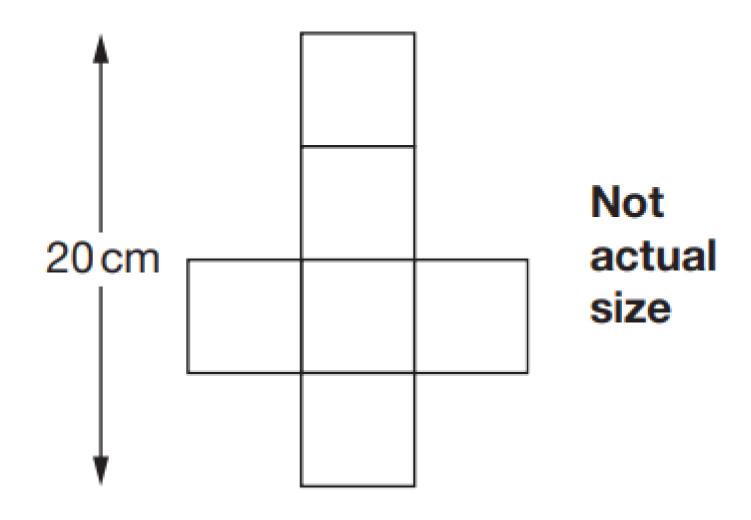
For each question, tick the correct answer. Then, explain the mistake.

- Linemburguise

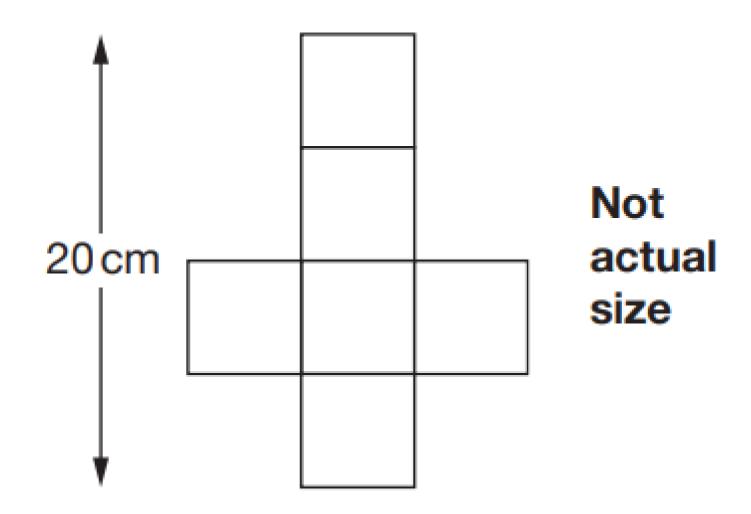
| Question   | Which answer? Explain the mistake.  |   |
|--|---|---|
| A band held a concert for charity.  Adult tickets = £15 Child tickets = £8  250 tickets were sold.  110 of these were child tickets. | Answer A:<br>$250 \times £15 = £3750$<br>$110 \times £8 = £880$<br>£3750 + £880 = £4630 | Answer B:<br>$140 \times £15 = £2100$<br>$110 \times £8 = £880$<br>£2100 + £880 = £2980 |
| How much money was raised?   | The mistake in A is they forgot to take the child tikets amounts off the adult          |   |
| 2000 people going to the match. Coaches can fit 60 people. Cars can fit 5 people. There are 25 coaches.                              | Answer A:<br>60 × 25 = 1500<br>2000 - 1500 = 500<br>500 cars needed                     | Answer B:<br>$2000 - 60 \times 25 = 500$<br>$500 \div 5 = 100$<br>100 cars needed       |
| How many cars are needed to take everyone to the match?  | They wrote the way  | nount of people   |



This is the net of a cube.



This is the net of a cube.



What is the **volume** of the cube?

How many does he eat in two years?

How many does he eat in two



How many does he eat in two years?

# How many does he eat in two years?

60 minutes = 1 hour

**24 hours = 1 day** 

7 days = 1 week

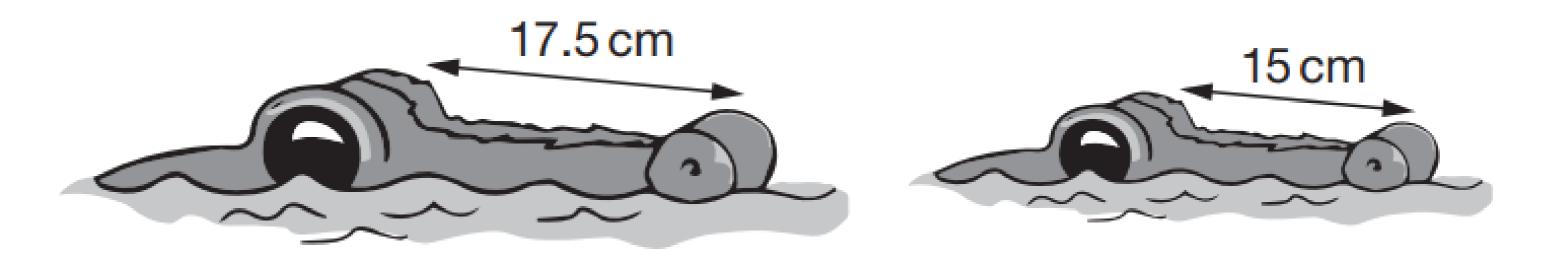
52 weeks ≈ 1 year

12 months = 1 year

365 days = 1 year (unless a leap year)

- measuring the distance from its eyes to its nose
- then multiplying that distance by 12

What is the difference in the estimated lengths of these two alligators?

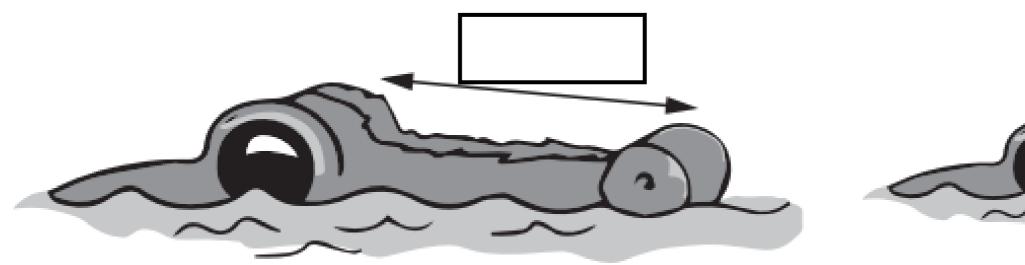


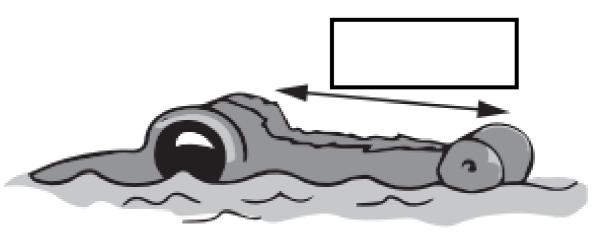
measuring the distance from its eyes to its nose

• then

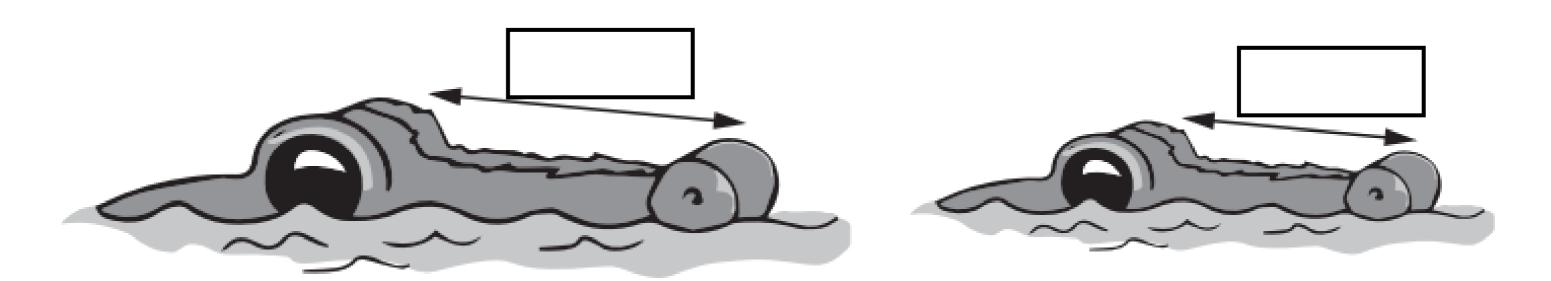
(a) Add 250cm

(b) Multiply by 12



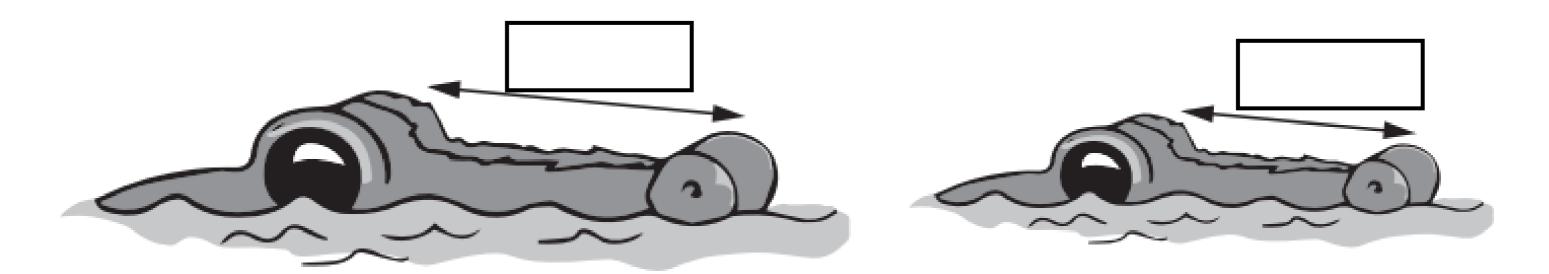


- measuring the distance from its eyes to its nose
- then multiplying that distance by 12



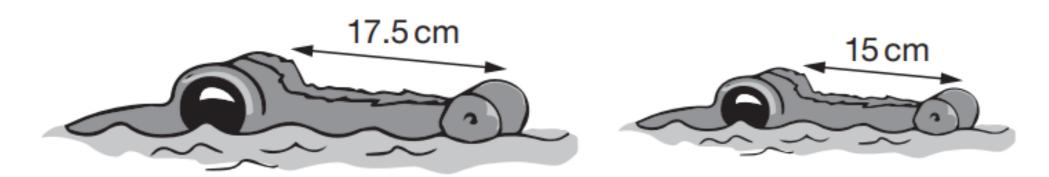
- measuring the distance from its eyes to its nose
- then multiplying that distance by 12

What is the difference in the estimated lengths of these two alligators?



- measuring the distance from its eyes to its nose
- then multiplying that distance by 12

What is the difference in the estimated lengths of these two alligators?



#### Not to scale

#### Method A:

 $17.5 \times 12 = 210$ cm

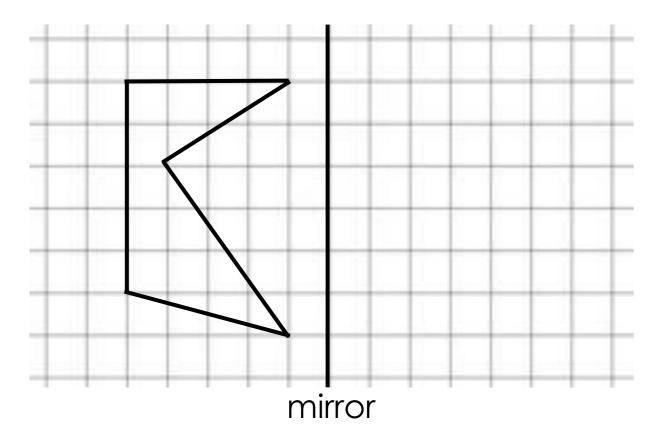
 $15 \times 12 = 180$ cm

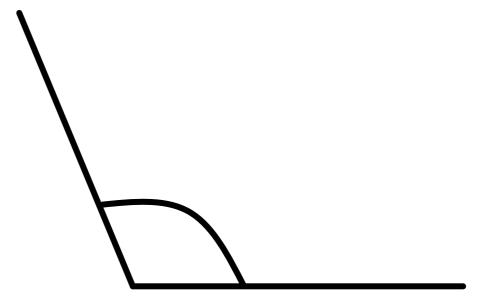
210cm - 180cm = 30cm

#### Method B:

17.5cm - 15cm = 2.5cm

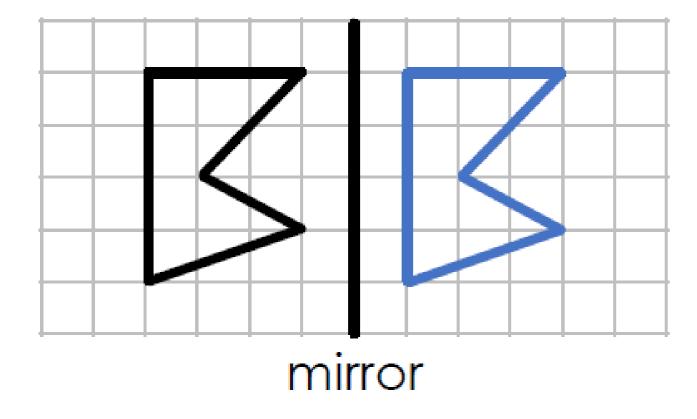
 $2.5 \text{cm} \times 12 = 30 \text{cm}$ 



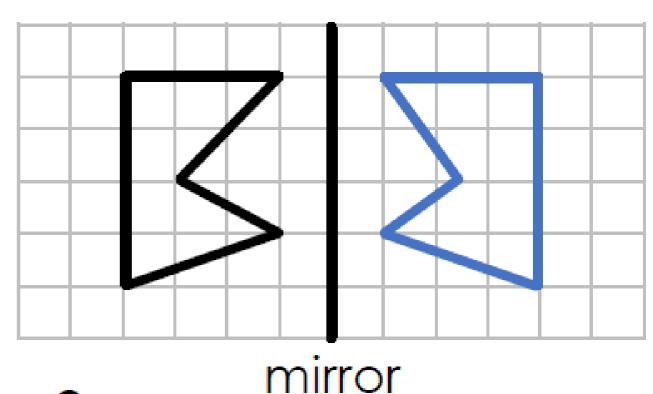


Reflect the shape in the mirror line.

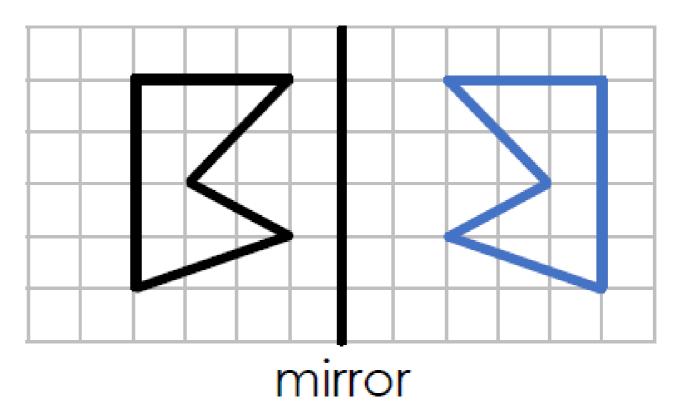
#### Mistake 1

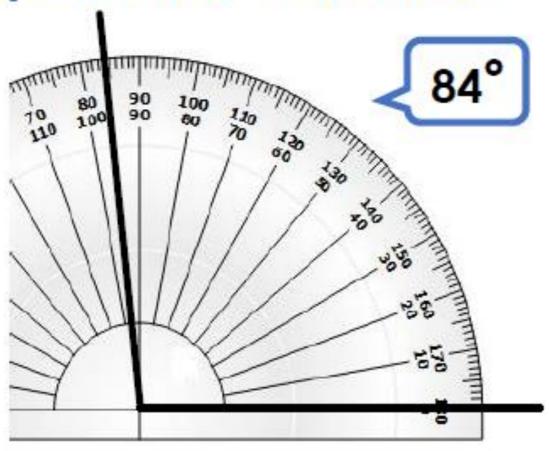


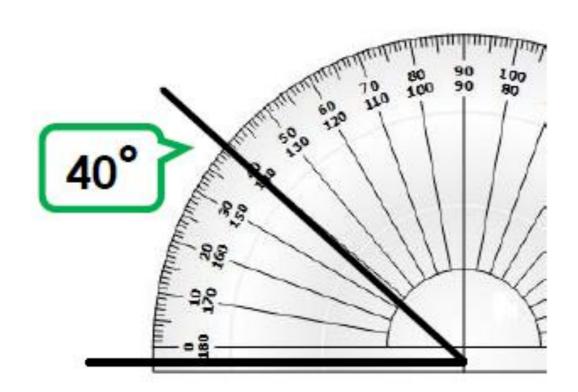
#### Mistake 3

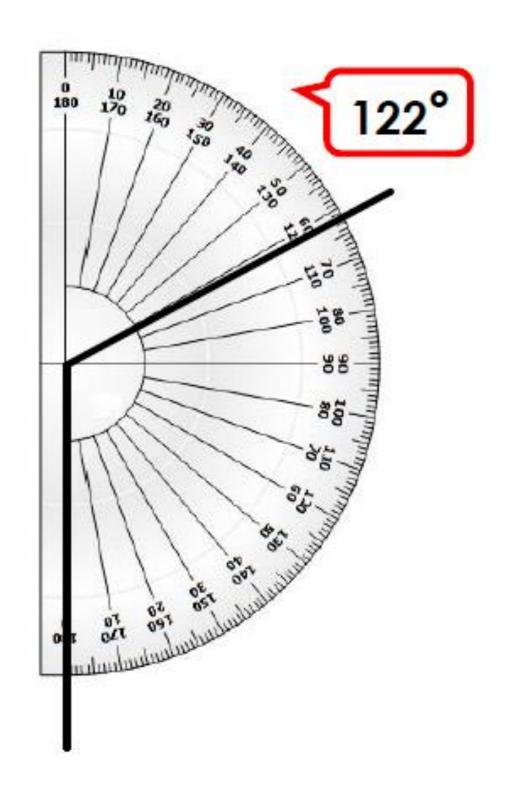


Mistake 2

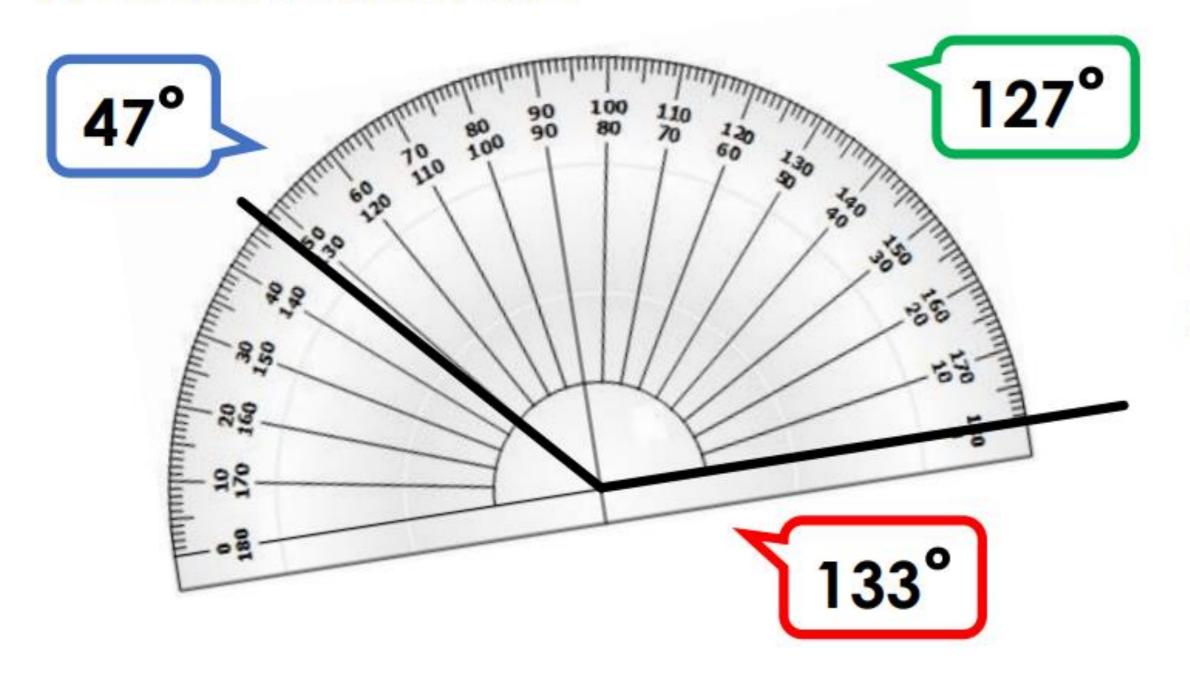








## Which Answer?



Explain the mistakes.

In Roman Numerals, 4 is written IIII

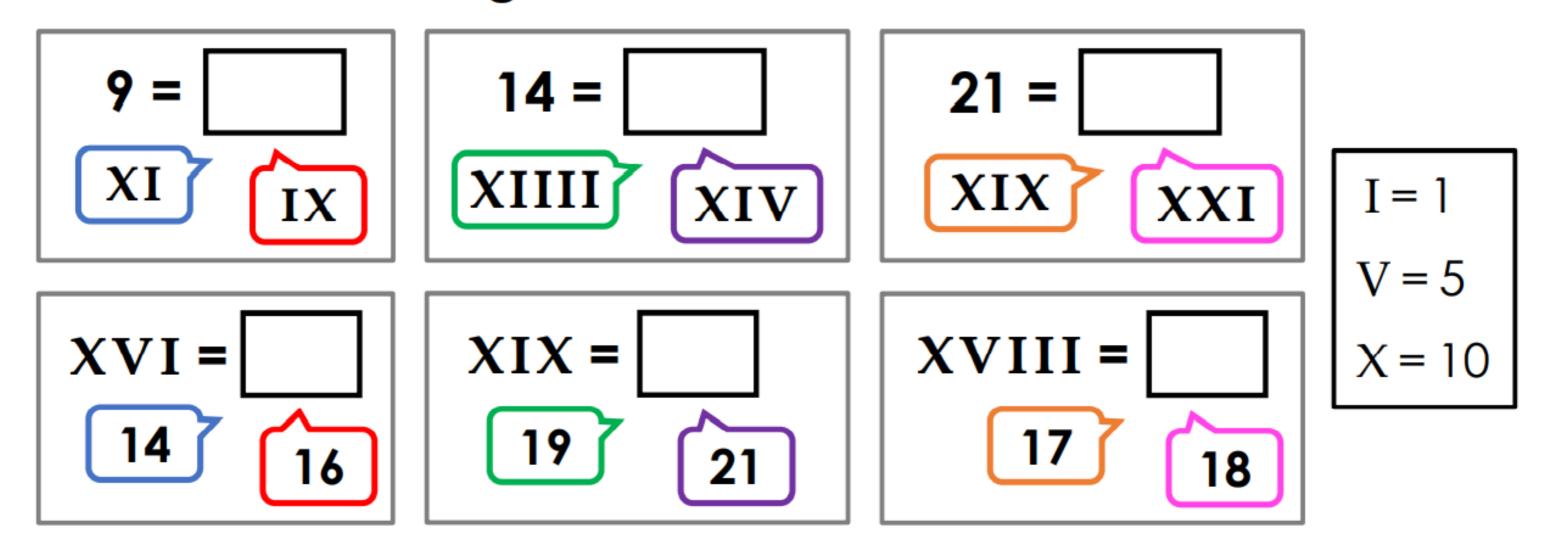
XL is 60 because it is 10 + 50

I = 1 V = 5 X = 10 L = 50

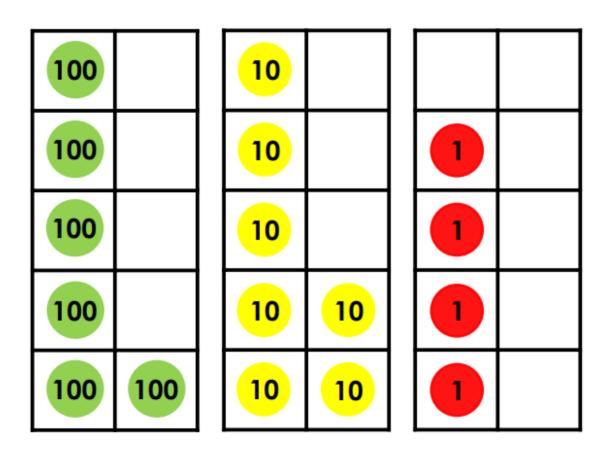
In Roman Numerals, 15 is written VVV

### Which Answer?

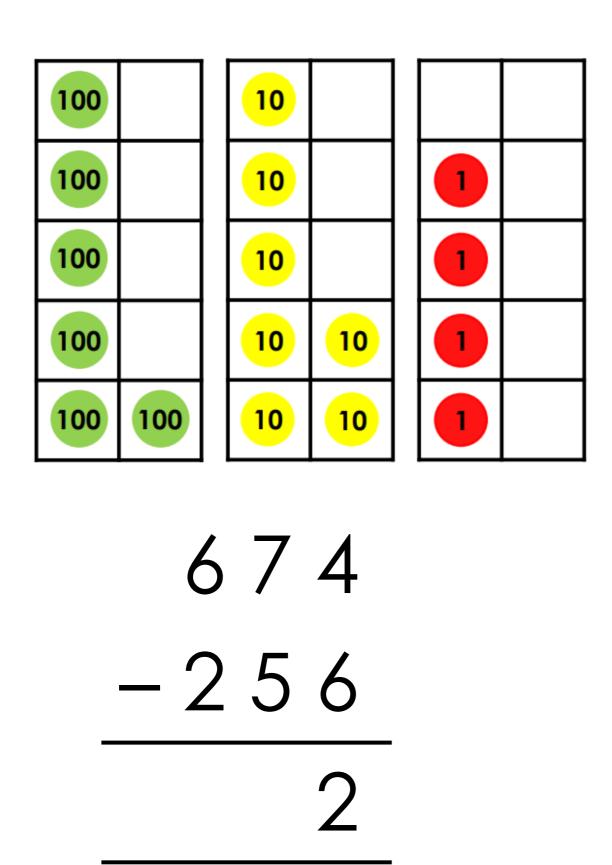
#### What is the missing Roman Numeral or number?



Explain the mistakes.



674 - 256



- 2 5 6

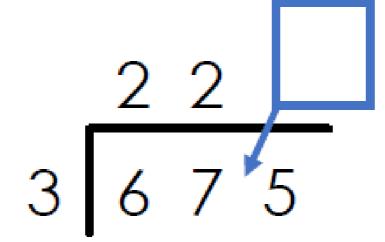
## Correct or Incorrect? $\checkmark$ or $\star$

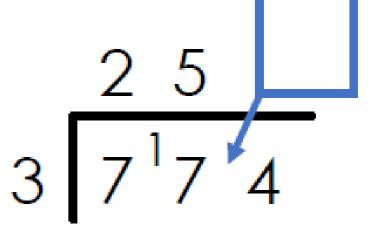


### Which of these regroups have been done correctly?

# **Next Step**

In each calculation, what's the remainder?





### Example A:

$$5 \ 2^{1}3 \ 3$$

$$-415$$

$$1083$$

### Example B:

$$\frac{1}{1}$$
  $\frac{1}{2}$   $\frac{6}{7}$   $\frac{7}{9}$   $\frac{2}{8}$   $\frac{8}{1}$ 

#### Example C:

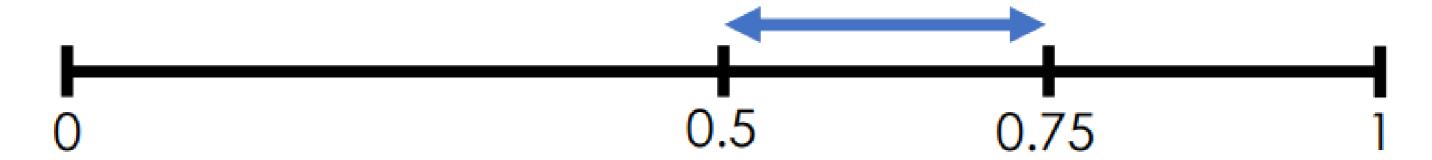
$$\begin{array}{r}
 8 \\
 8 \\
 0^{1}2 \\
 3 \\
 -6 \\
 07 \\
 0 \\
 \hline
 20 \\
 53 \\
 \end{array}$$

**Extend:** complete the calculations correctly.

## **How Many Ways?**

Make all the fractions that are more than 0.5 and less than 0.75 using these numbers:

2 3 4 5 6 8



Level 1: I can find a way

Level 2: I can find different ways

Level 3: I know how many ways there are

# **How Many Ways?**

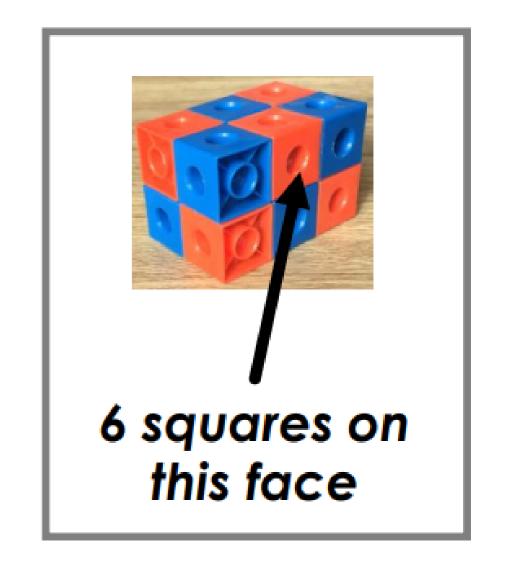
Make a cuboid using 16 to 18 cubes.

There must be at least 4 squares on each face of the cuboid.

Level 1: I can find a way

Level 2: I can find different ways

Level 3: I know how many ways there are



### **How Many Ways?**

$$\frac{\square}{8} + \frac{\square}{\square} = \frac{\square}{4}$$

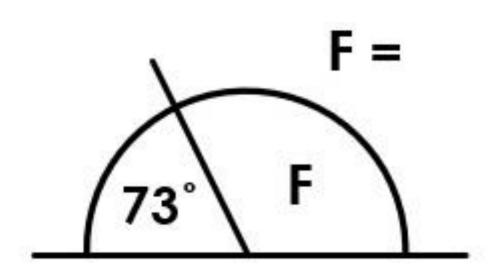
The answer must be a proper fraction.

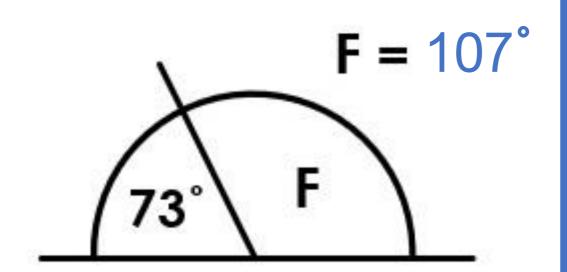
Level 1: I can find a way

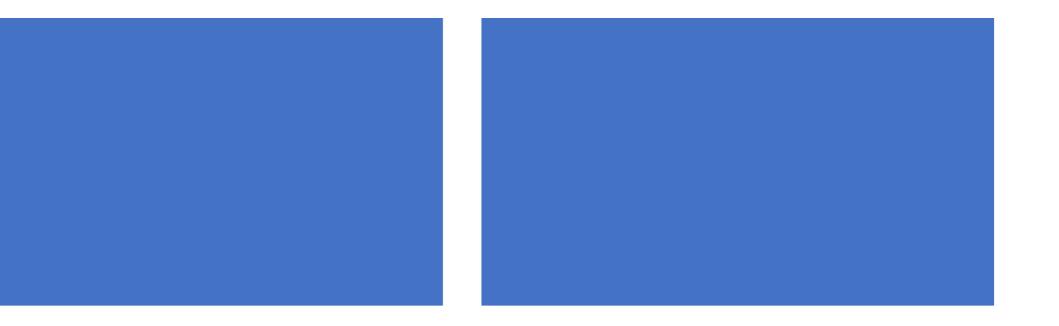
Level 2: I can find different ways

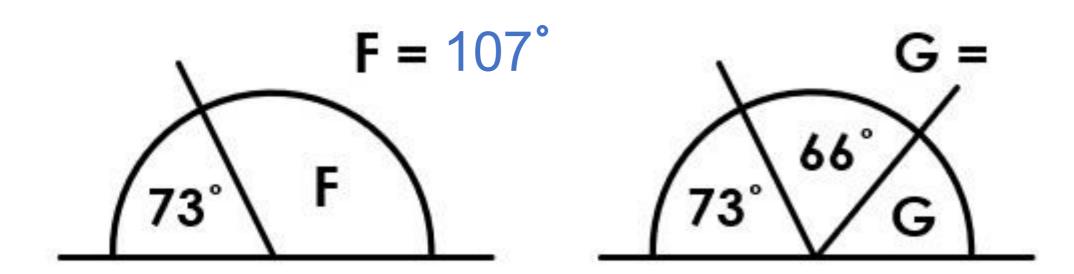
Level 3: I know how many ways

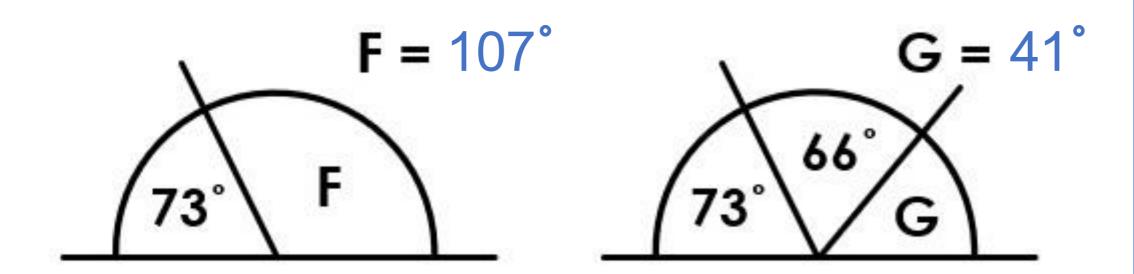
there are

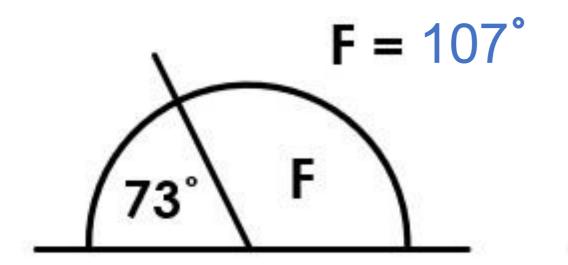


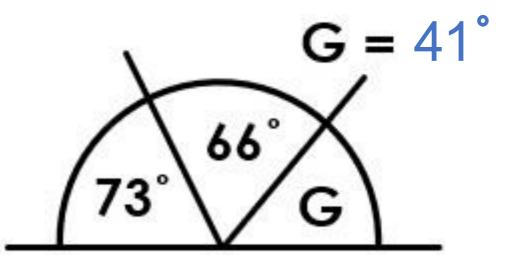


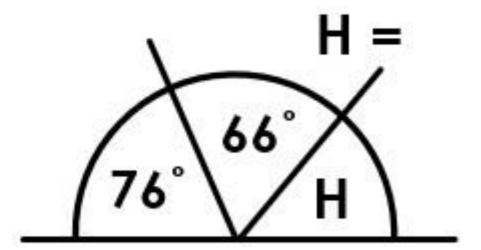


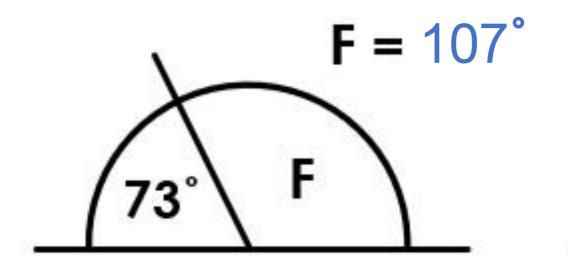


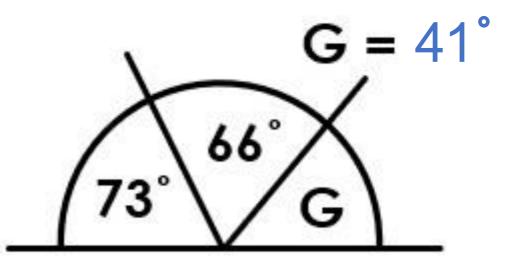


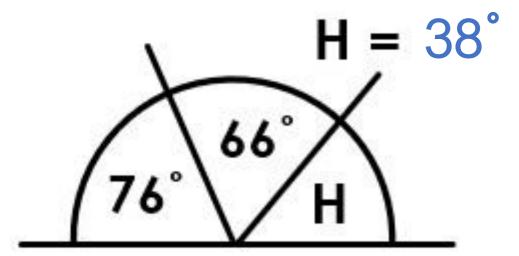


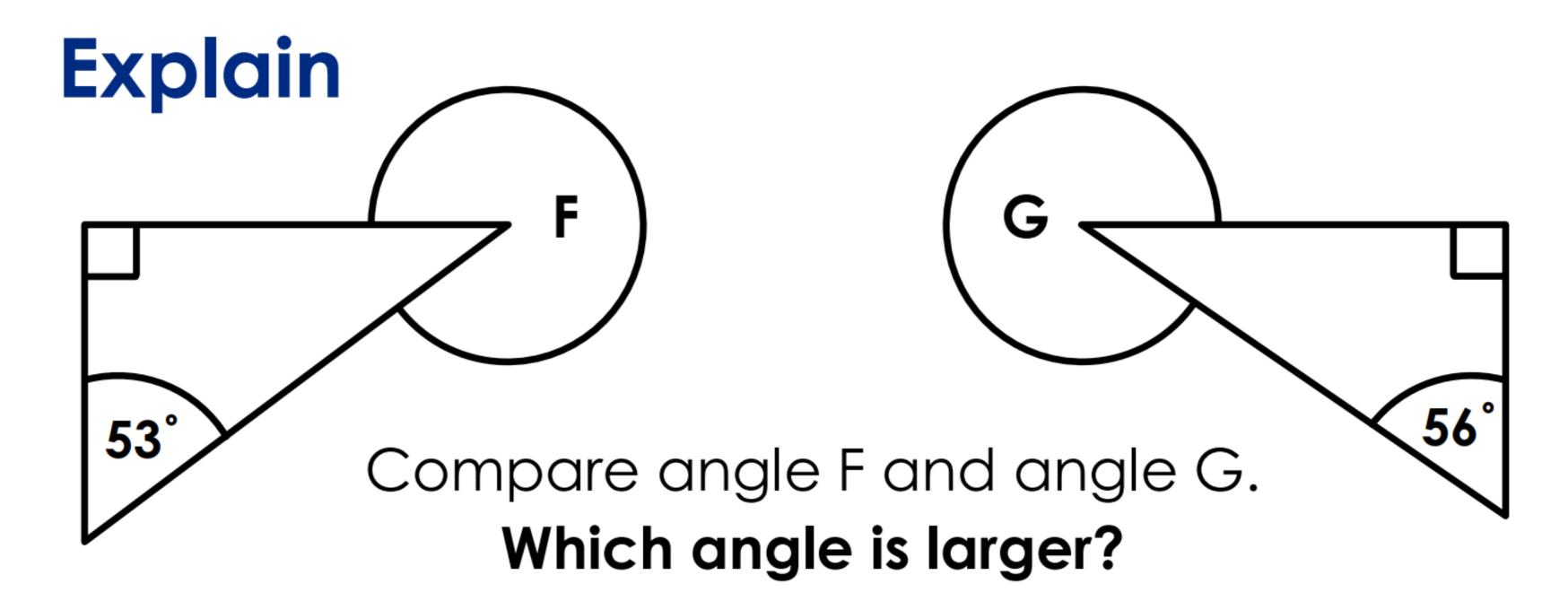












4, \_\_\_\_, 22

4, \_\_\_\_, 22

4, 13, 22

4, 13, 22

4, 10, 16, 22

$$100 - (10 \times 6) =$$

$$(100 - 10) \times 6 =$$

$$100 - 10 \times 6 = 40$$

$$100 - (10 \times 6) =$$

$$(100 - 10) \times 6 =$$

$$100 - (10 \times 6) = 40$$

$$(100 - 10) \times 6 =$$

$$100 - 10 \times 6 = 40$$
 $100 - (10 \times 6) = 40$ 
 $(100 - 10) \times 6 = 540$ 

$$\frac{1}{10}$$
 of 40 =

$$\frac{1}{5}$$
 of 40 =

$$\frac{2}{5}$$
 of 40 =

$$\frac{4}{10}$$
 of 40 =

$$\frac{1}{10}$$
 of  $40 = 4$ 

$$\frac{1}{5}$$
 of 40 = 8

$$\frac{2}{5}$$
 of  $40 = 16$ 

$$\frac{4}{10}$$
 of  $40 = 16$ 

(d) 364 + 183

(e) 381 + 166

(d) 674 - 385

(e) 551 - 262

(d)  $342 \times 8$ 

(e)  $684 \times 4$ 

(d)  $162 \div 3$ 

(e)  $324 \div 6$ 

$$162 \div 3$$

$$324 \div 6$$

$$694 - 365$$

$$674 - 385$$

$$551 - 262$$

$$\frac{1}{10}$$
 of 40 =

$$\frac{1}{5}$$
 of 40 =

$$\frac{2}{5}$$
 of 40 =

$$\frac{4}{10}$$
 of  $40 =$ 

$$28 \div 4 = \times 1$$

92. In some schools, pupils were not explicitly taught how to apply the mathematics they had recently learned to mathematical problems. Their only exposure to solving mathematical problems was through answering the final few questions of a predominantly procedure-focused exercise. Often, many pupils did not reach this stage of the exercise. These pupils, therefore, had very little experience of applying mathematical methods beyond routine and established applications. Pupils in these schools were notably less confident when solving mathematical problems.

93. In a minority of schools, problem-solving was explicitly planned into the curriculum. Teachers understood the importance of demonstrating how to apply mathematical methods to problems and giving pupils multiple opportunities to practise applying these methods to structurally similar problems. In the most successful lessons, teachers clearly 'drew out' the similarities between problems to help pupils identify the mathematical techniques that might be useful for different types of problem.



### Contexts

### Which questions are answered by calculating $15 \div 5$ ?

- (a) There are 15 children in the hall. There are 5 children at each table. **How many tables are there in the hall?**
- (b) 15 people have 5 grapes each. How many grapes in total?
- (c) 5 children share 15 pencils. How many pencils each?
- (d) Of the 15 children that go to running club, there are 5 girls. How many boys go to running club?

# Interpreting Remainders

- (a) Cupcakes are packed in boxes of 6. The bakery has made 30 cupcakes. How many full boxes can be made?
- (b) Cupcakes are packed in boxes of 6. The bakery has made 32 cupcakes. How many full boxes can be made?
- (c) Cupcakes are packed in boxes of 6.
  The bakery has made 32 cupcakes.
  How many boxes are needed to hold all of the cupcakes?

Tom had

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

Tom had

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

**Money Tom had** 

Tom had

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

**Money Tom had** 

£30

Tom had

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

Money Tom had £30

Cost of bike £60 left

Tom had

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

| Money Tom had |          | £30 |
|---------------|----------|-----|
| £60 bike      | £60 left |     |

Tom had £90

He was given £30 for his birthday.

Then he spent **half** of his money on a bike.

Tom has £60 left.

| Tom had £90 |          | £30 |
|-------------|----------|-----|
| £60 bike    | £60 left |     |

My number was

I multiply my number by 4

Then I subtract 15

Build 1

My number was

I multiply my number by 4

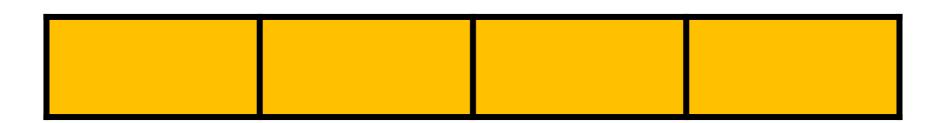
Then I subtract 15



My number was

I multiply my number by 4

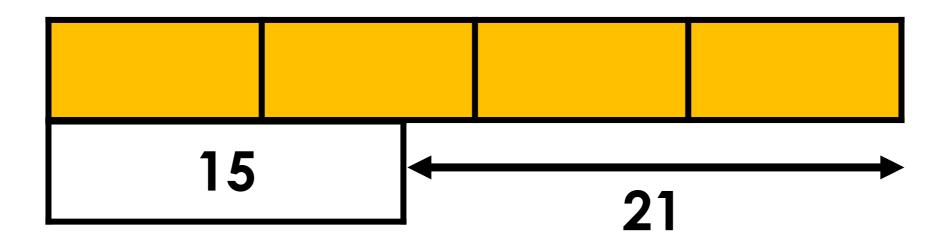
Then I subtract 15



My number was

I multiply my number by 4

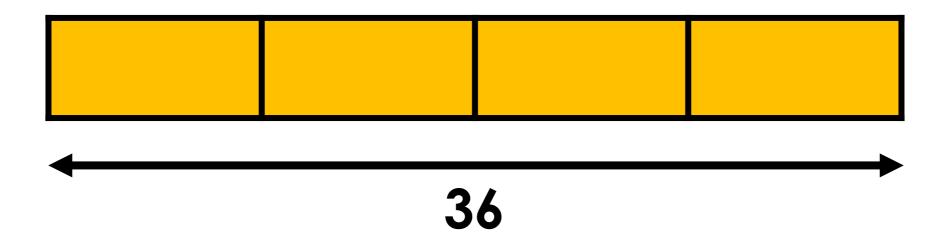
Then I subtract 15



My number was

I multiply my number by 4

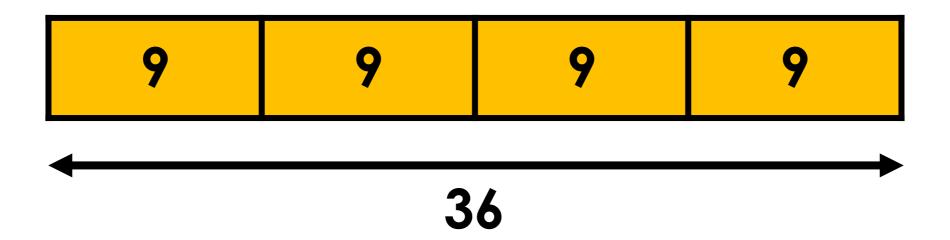
Then I subtract 15



My number was 9

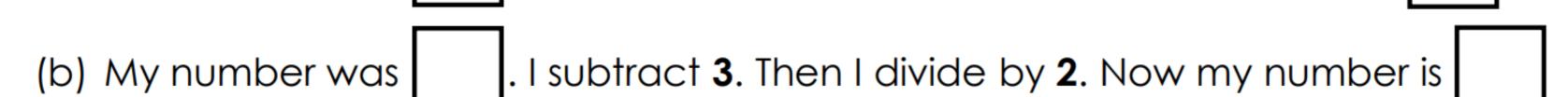
I multiply my number by 4

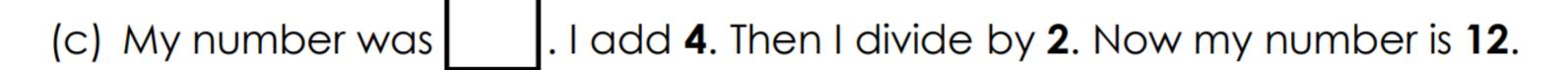
Then I subtract 15



Inverse

(a) My number was . I add 3. Then I multiply by 4. Now my number is





(d) My number was . I add 4. Then I multiply by 2. Now my number is 12.

(e) My number was **15**. I multiply by **4**. Then I subtract **25**. Now my number is

(f) My number was . I add **25**. Then I divide by **4**. Now my number is **15**.





Jen thinks of a number.

She multiplies her number by 3

Then she adds 2

Now Jen's number

IS

She adds 2

Then she multiplies

her number by 3

Now Jen's number

IS

'The blue/red number will be larger because...'



# Jen thinks of a number.



Then she adds 2

Now Jen's number İS

She adds 2

Then she multiplies

her number by 3

Now Jen's number

IS





Jen thinks of a number.



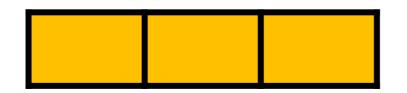
Then she adds 2

Now Jen's number İS

# She adds 2

Then she multiplies her number by 3

Now Jen's number İS





Inverse Build 2

Jen thinks of a number.



Then she adds 2

Now Jen's number is

She adds 2

Then she multiplies her number by **3** 

Now Jen's number is





Jen thinks of a number.



number by 3

Then she adds 2

Now Jen's number

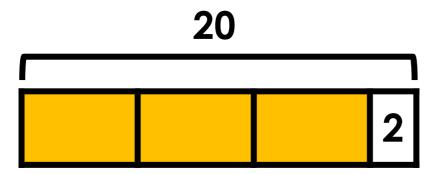
is **20** 

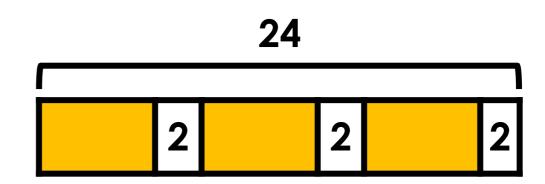


Then she multiplies

her number by 3

Now Jen's number





Inverse Build 2

Jen thinks of a number.



number by 3

Then she adds 2

Now Jen's number

is **20** 

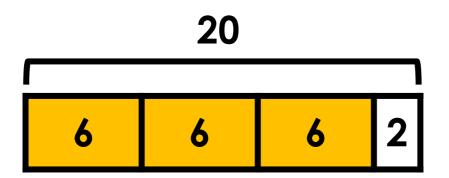
She adds 2

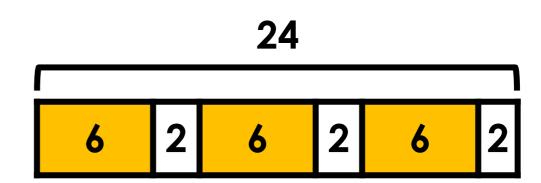
Then she multiplies

her number by 3

Now Jen's number

**24** 





- (a) I chose a number. I multiplied my number by 3. Then I added 5. Now, my number is 26. What number did I choose? 7
- (b) I chose a number. I multiplied my number by 3. Then I added 6. Now, my number is 27. What number did I choose? 7
- (c) I chose a number. I multiplied my number by 3. Then I subtracted 6. Now, my number is 27. What number did I choose?
- (d) I chose a number. I divided my number by 3. Then I subtracted 6. Now, my number is 27. What number did I choose?  $\P$

The answer to question is the same larger smaller than question because...

My number was

| multiply/divide my number by 6

add/subtract 3

Now my number is 27

What is the largest number that could go in the blue box? 18

What is the smallest number that could go in the blue box?

To make the number in the blue box as large as possible...

- and -, the start must be as big as possible

**Build 1** 

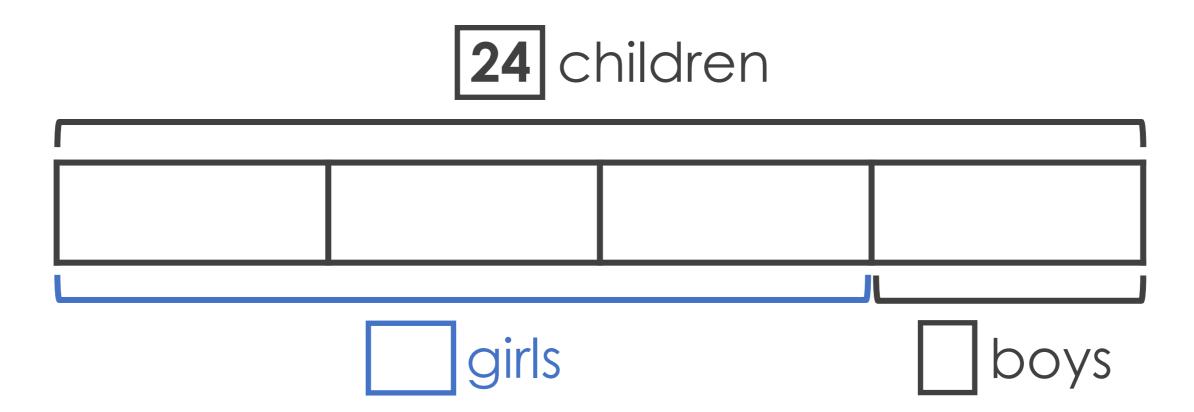
There are 24 children at running club.

 $\frac{3}{4}$  of the children at running club are girls.

**Build 1** 

There are 24 children at running club.

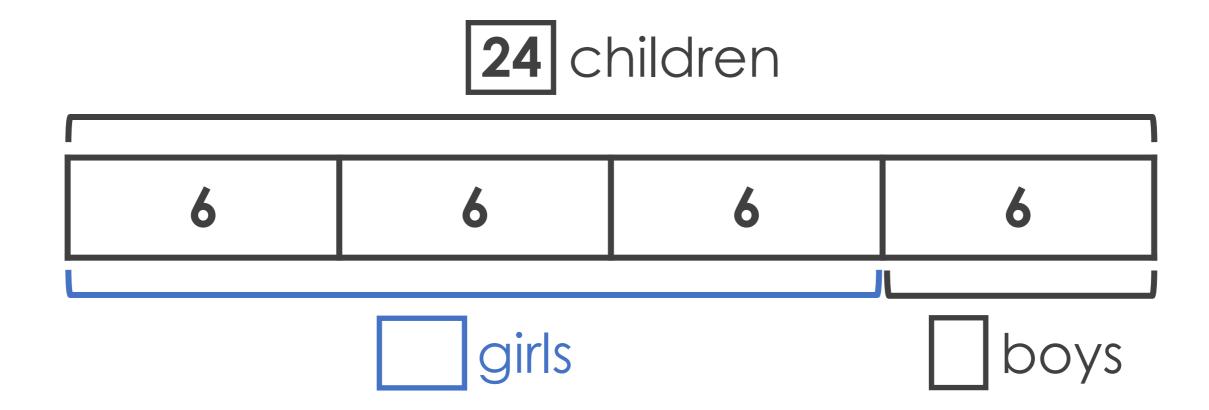
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**Build 1** 

There are 24 children at running club.

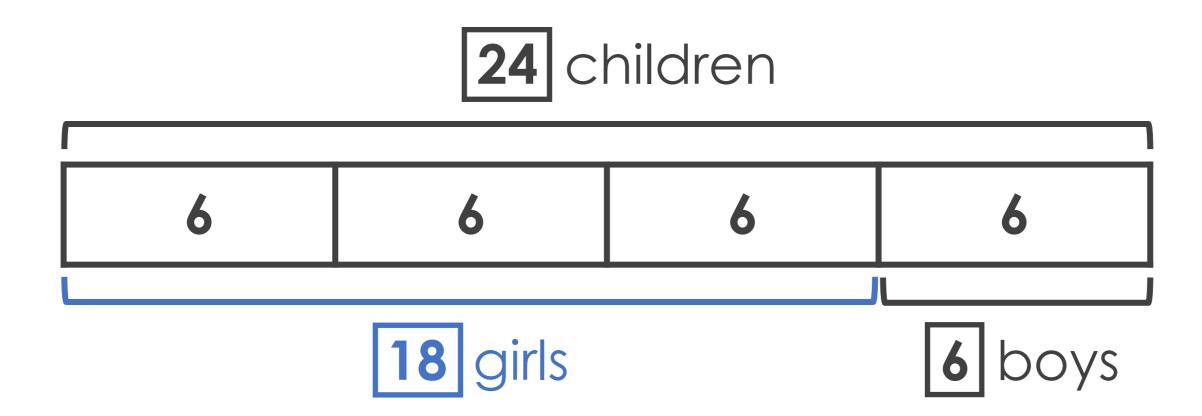
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**Build 1** 

There are 24 children at running club.

 $\frac{3}{4}$  of the children at running club are girls.



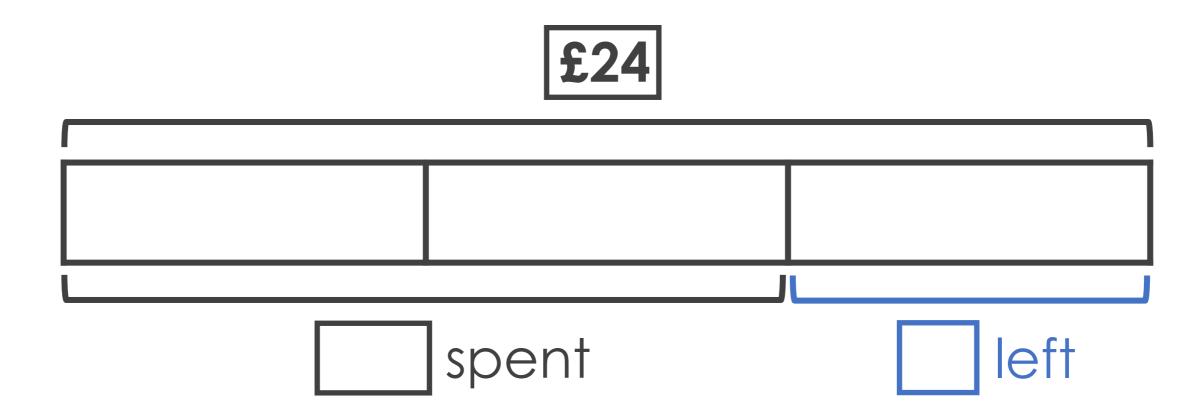
Ben had £24.

Then, he spent  $\frac{2}{3}$  of his money on a t-shirt.

**Build 1** 

Ben had £24.

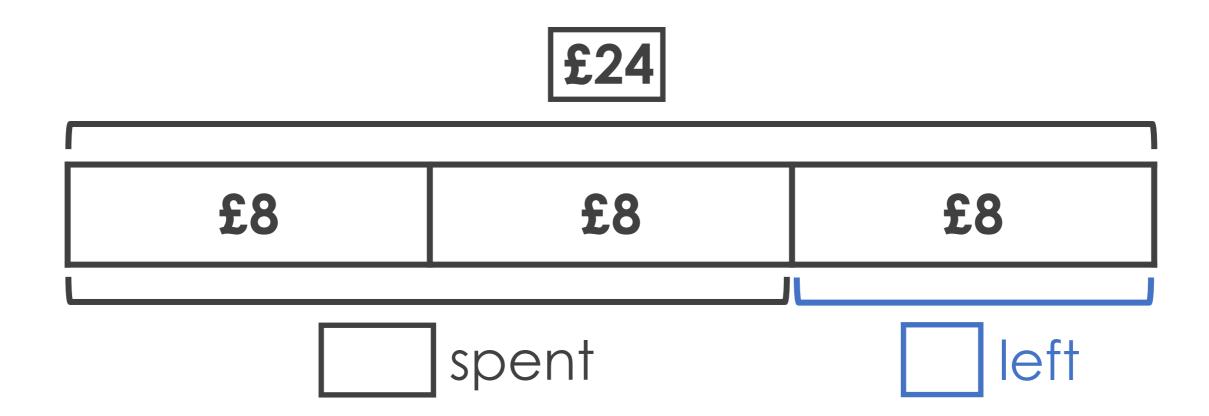
Then, he spent  $\frac{2}{3}$  of his money on a t-shirt.





Ben had £24.

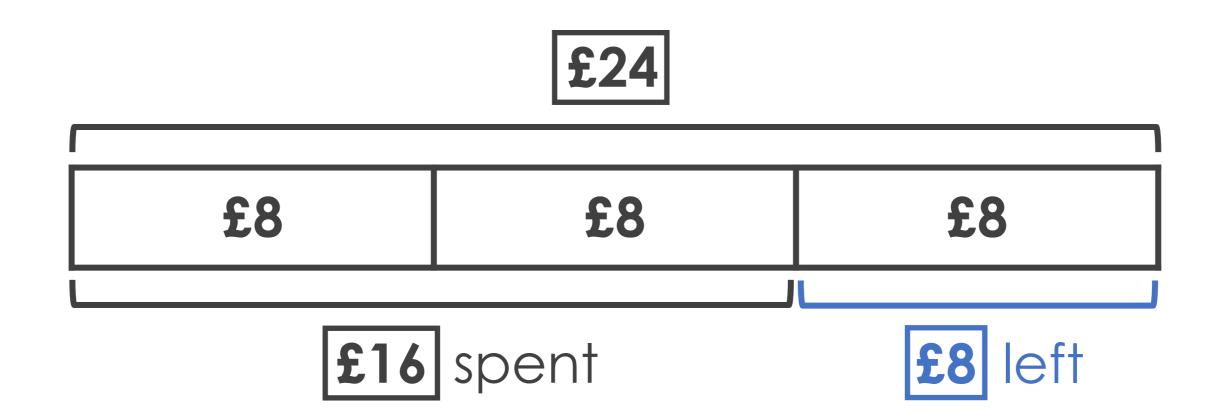
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Ben had £24.

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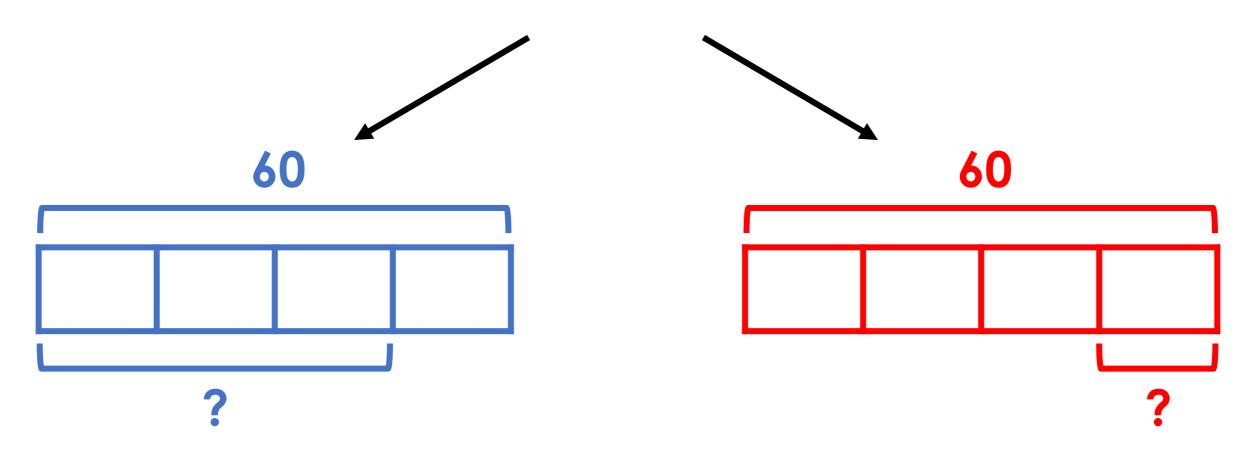


Zara's book is 60 pages long.

Zara has read  $\frac{3}{4}$  of her book.

How many pages does Zara have left to read?

Which bar model represents the question?



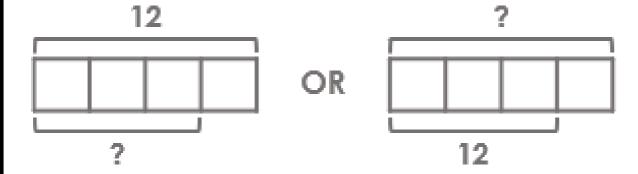
### Task A

Which bar model correctly represents each question?

There are 12 children at the beach.

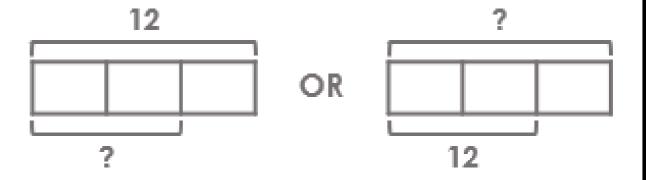
 $\frac{3}{4}$  of these children are girls.

How many girls at the beach?



Sam plays tennis. This year, he won  $\frac{2}{3}$  of his matches. Sam won 12 matches.

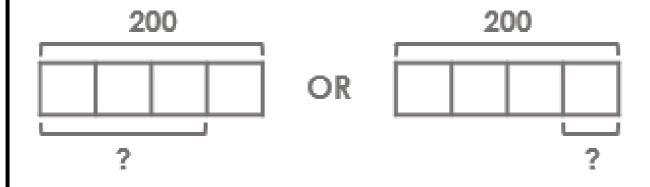
How many tennis matches did Sam play?



Lucy's book is 200 pages long.

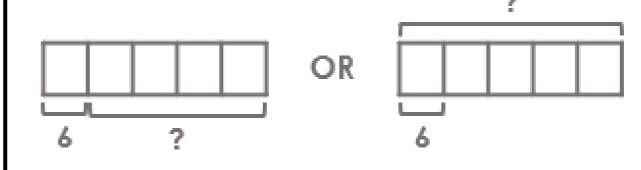
Lucy has read  $\frac{3}{4}$  of her book.

How many pages does Lucy have left?

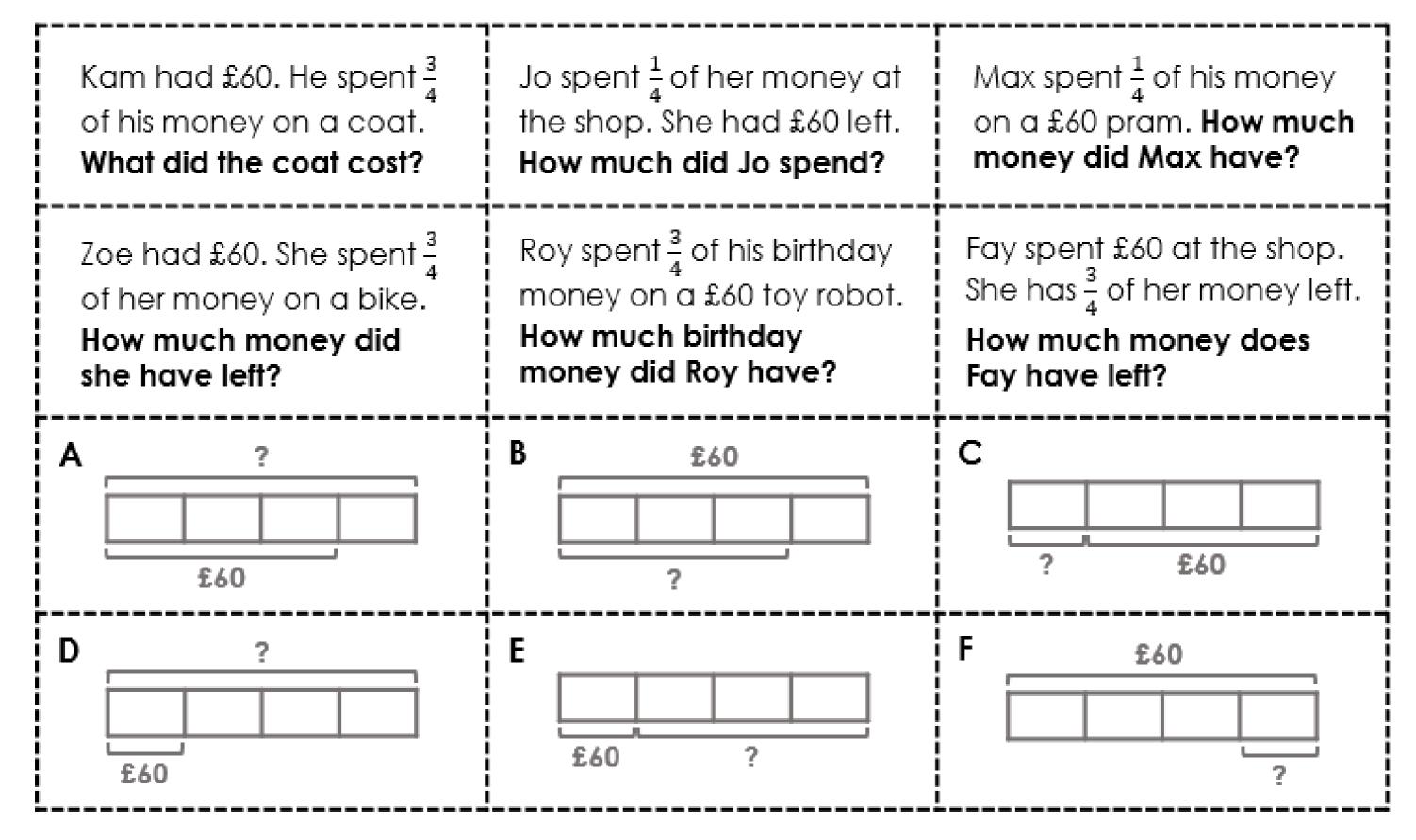


6 children in the class are left-handed. This is  $\frac{1}{5}$  of the children in the class.

How many right-handed children in the class?



# Cut out. Match each question to the appropriate bar model.



| Question  | Complete the bar model and answer: |  |
|---|------------------------------------|--|
| (a) Tom has £30. He spends $\frac{3}{5}$ of his money on a toy.  How much does the toy cost?              | £30                                |  |
| (b) There are 30 coins on the table. $\frac{3}{5}$ are showing heads. How many are showing tails?         | heads                              |  |
| (c) Jo has read $\frac{3}{5}$ of her book.<br>She has read 30 pages.<br>How many pages long is Jo's book? |                                    |  |

Task D

- (a)  $\frac{1}{5}$  of 420 = (b)  $\frac{1}{6}$  of 420 =
- (c)  $\frac{1}{7}$  of 420 =
- (d) 6 plums weigh 420g. Zack eats one plum. What is the weight of the remaining plums?
- (e)  $\frac{5}{6}$  of the 420 children in the school are right-handed. How many left-handed children are there in the school?
- (f) Joy has read  $\frac{2}{5}$  of her 420-page book. How many pages does she have left to read?
- (g) Adam used  $\frac{5}{7}$  of the sugar from the pack in the cake. There were 420g sugar left in the pack.

How many grams of sugar are there in the cake?

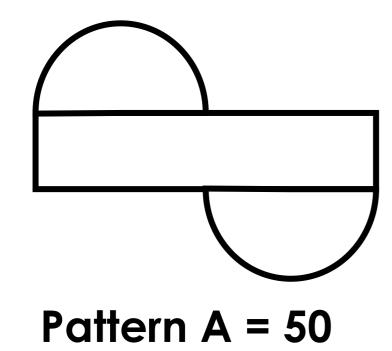
(h) For every 5 women at the concert, there were 2 men. There were 420 men at the concert.

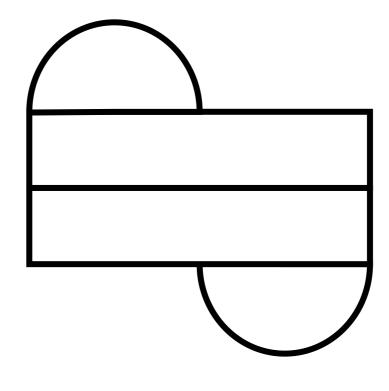
How many women were there at the concert?

**Extension:** Compare questions (g) and (h).



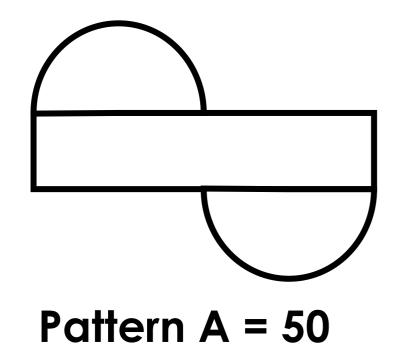
The patterns are made with identical rectangles and semi-circles.

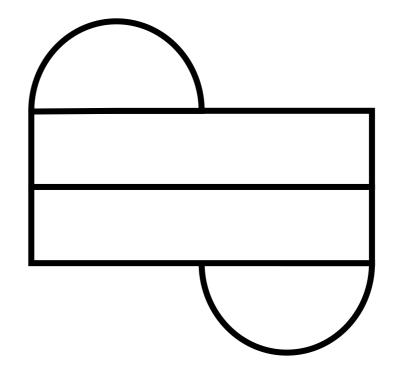




Pattern B = 70

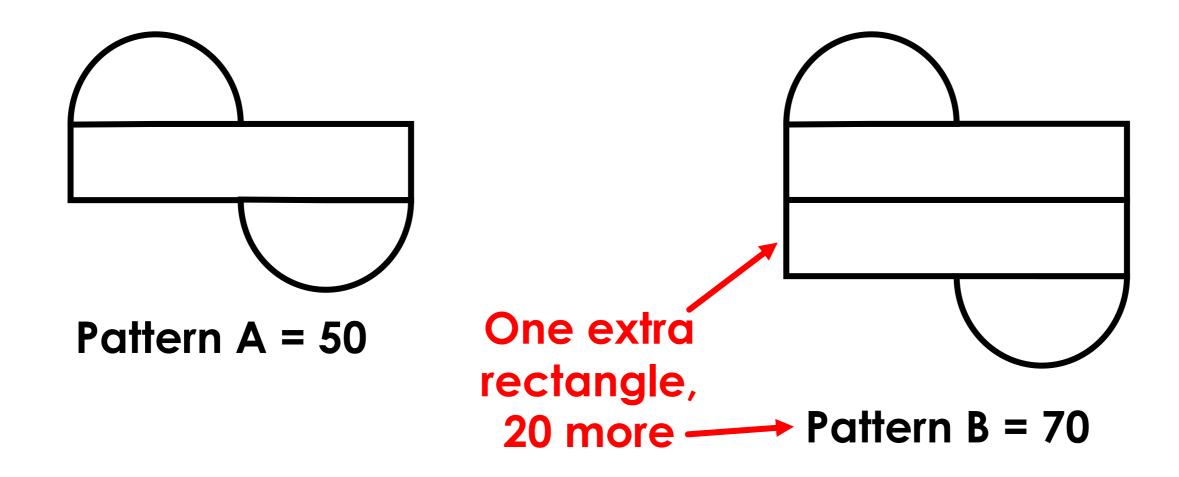


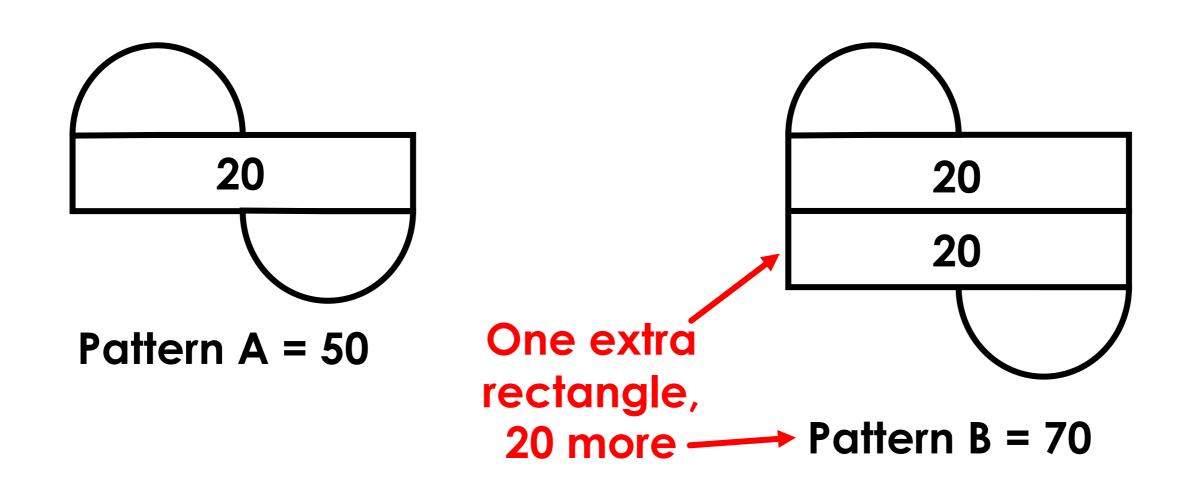


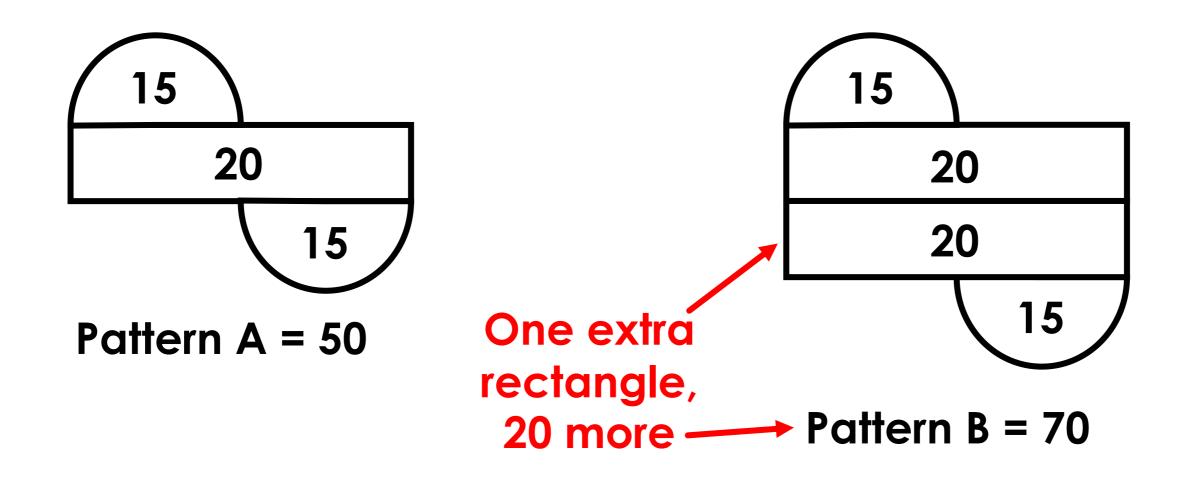


Pattern B = 70

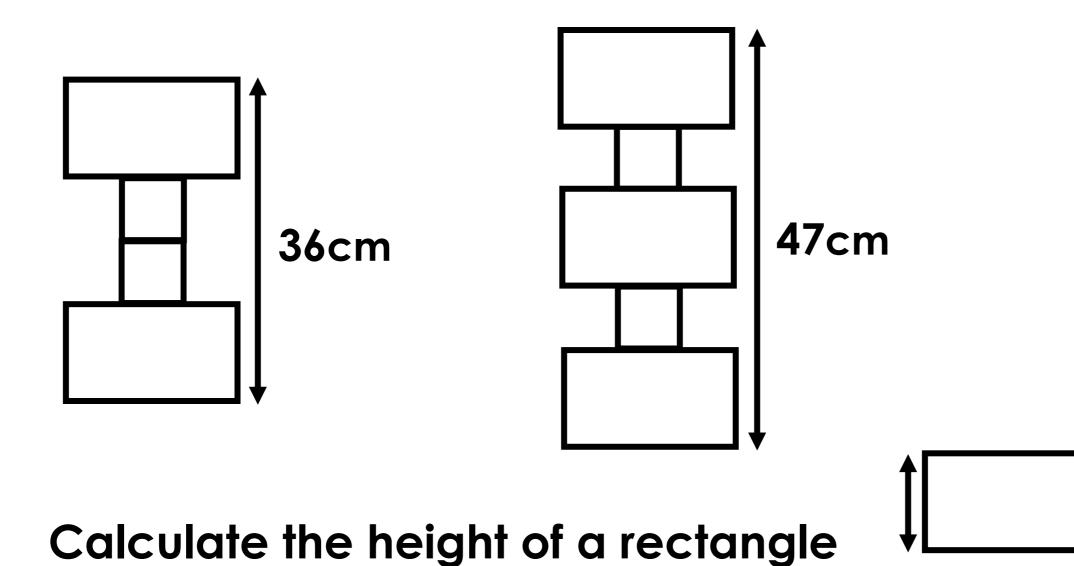








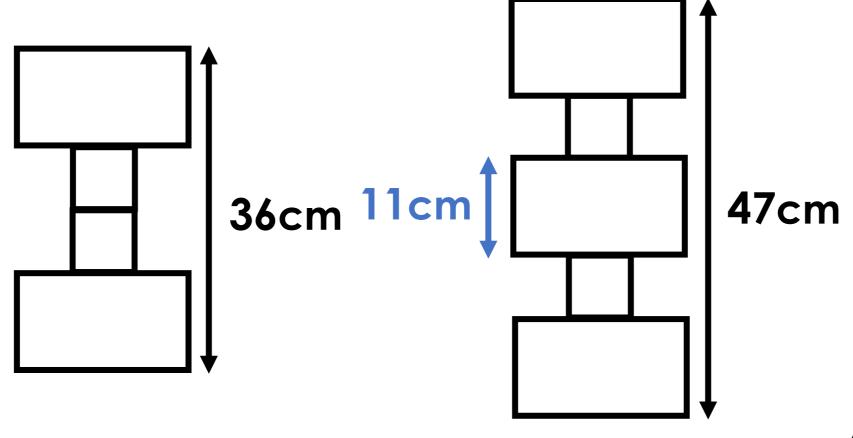
The towers are made with identical squares and identical rectangles.



and the height of and a square.



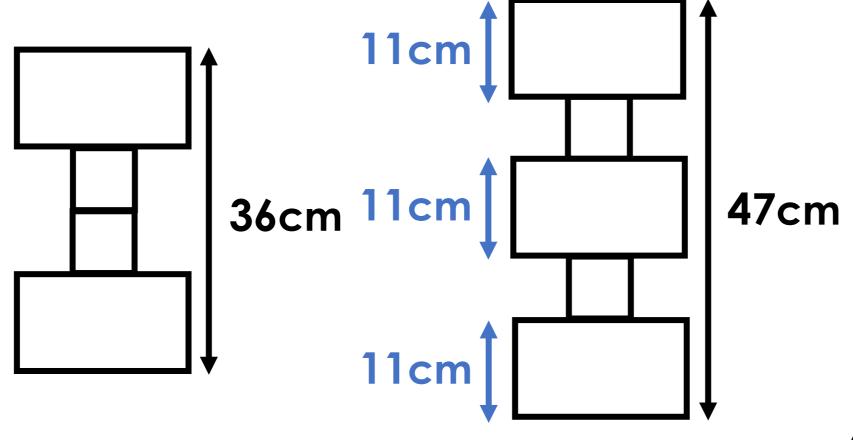
The towers are made with identical squares and identical rectangles.



Calculate the height of a rectangle and the height of and a square.

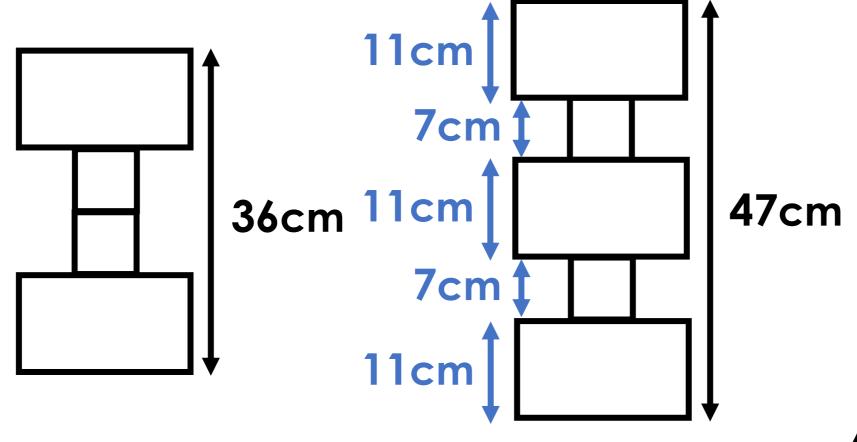


The towers are made with identical squares and identical rectangles.



Calculate the height of a rectangle and the height of and a square.

The towers are made with identical squares and identical rectangles.



Calculate the height of a rectangle and the height of and a square.

3 adults and 2 children go to the show.

They pay for their tickets.

2 adults and 2 children go to the show.

They pay for their tickets.

What is the cost of a child ticket to the show?

Child ticket =



3 adults and 2 children go to the show.

They pay £58 for their tickets.

2 adults and 2 children go to the show.

They pay £44 for their tickets.

What is the cost of a child ticket to the show?

Child ticket =

3 adults and 2 children go to the show.

They pay £58 for their tickets.

2 adults and 2 children go to the show.

They pay **£44** for their tickets.

What is the cost of a child ticket to the show?







$$= £58$$





$$= £44$$

**Build 2** 

3 adults and 2 children go to the show.

They pay **£58** for their tickets.

2 adults and 2 children go to the show.

They pay **£44** for their tickets.

What is the cost of a child ticket to the show?

$$\triangle = £14$$







$$c$$
 = £58

£14



3 adults and 2 children go to the show.

They pay **£58** for their tickets.

2 adults and 2 children go to the show. They pay **£44** for their tickets.

What is the cost of a child ticket to the show?

$$\triangle = £14$$







$$c = £58$$

£14

$$=$$
£44

3 adults and 2 children go to the show. They pay £58 for their tickets.

2 adults and 2 children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show?

$$\triangle = £14$$

$$c = £8$$



#### Question

#### Complete the bar model and answer:

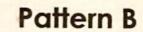
3 apples and 2 pears cost £1.40

2 apples and 2 pears cost £1.10

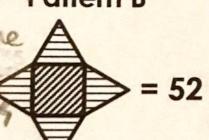
What is the cost of a pear?

Clue: Start by calculating the cost of an apple

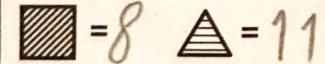
Pattern A







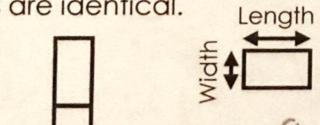
rename



Clue: Pattern B has 1 more triangles than Pattern A

The rectangles are identical.

33cm 9



Width = 
$$\delta$$

Choose: Start by calculating the length OR width

| Question  | Complete the barmodel and answer   |
|---|--|
| 3 apples and 2 pears cost £1.56 2 pears and 1 apple costs 84p What is the cost of a pear? | 7849 (D=36)=24<br>84p °156 72p   |
| Pattern A Pattern B = 75 = 75   | 75<br>-24<br>-24<br>-17×3=51<br>24-17=7  |
| The rectangles are identical.  Length  Length = 8.5 cm  Width = 5.5 cm                    | 17-2-8.5 233.5 35.5<br>17.0 3165 133<br>+0885 -17.0 3165 133<br>+0885 -17.0 3165 |

There are 28 pupils in a class.

The teacher has 8 litres of orange juice.

She pours 225 millilitres of orange juice for every pupil.



How much orange juice is left over?