## Creating Reasoning Routines, Building Problem-Solvers <br> Session 2

Reasoning throughout knowledge acquisition
www.iseemaths.com
Y5 \& Y6

## Routines Within Interactive Teaching

## Raising the internal narrative:

Gap between question and response/discussion, silent modelling

## Mass participation: <br> Form of answer before question, routines

## Managing discussions:

Quality modelling + participation, wait time 2

## Calculate the area of the shape:



## Calculate the area of the shape:



$$
3 \mathrm{~cm} \times 10 \mathrm{~cm}=30 \mathrm{~cm}^{2}
$$

## Calculate the area of the shape:



$$
\begin{aligned}
& 3 \mathrm{~cm} \times 10 \mathrm{~cm}=30 \mathrm{~cm}^{2} \\
& 7 \mathrm{~cm} \times 9 \mathrm{~cm}=63 \mathrm{~cm}^{2}
\end{aligned}
$$

| $\frac{3}{8}+\frac{2}{8}=\left(\frac{5}{8}\right)$ | $\frac{2}{16}+\frac{6}{16}$ <br> $\left(\frac{2}{8}\right)+\left(\frac{3}{8}\right)$$=\frac{5}{8}$ |
| :---: | :---: |$|$| $\frac{\text { Non }}{\frac{1}{2}+\frac{2}{16}}=\frac{5}{8}$ |
| :---: |
| $\left(\frac{4}{8}\right) \cdot\left(\frac{1}{8}\right)$ |

There are 20 children at the park.
There are 4 times as many girls as boys at the park. How many boys at the park?

There are 20 children at the park.
There are 4 times as many girls as boys at the park. How many boys at the park?

$\frac{1}{4}$ of a number is 20 .
What is the number?

## $\frac{1}{4}$ of a number is 20 . What is the number?



Ken thinks of a number. He divides it by 3 . The answer is 72 .

## What number was Ken thinking of?

Ken thinks of a number. He divides it by 3 . The answer is 72 .

## What number was Ken thinking of?



## $\frac{1}{4}$ of $\square=\square$






## $\frac{2}{3}$ of $\square=\square$

## $\frac{2}{3}$ of $\square=\square$




## $\frac{2}{3}$ of $\square=12$



A circus is holding a concert for charity.
Adult tickets cost $\mathbf{£ 1 1}$. Child tickets cost $\mathbf{£ 6}$.

## How many child tickets are sold?

What information must be given?

A circus is holding a concert for charity.
Adult tickets cost $£ \mathbf{1 1}$. Child tickets cost $\mathbf{£ 6}$.
$\mathbf{1 2 0}$ adult tickets are sold. In total, $\mathbf{£ 1 8 0 0}$ is raised.
How many child tickets are sold?

For each question, tick the correct answer. Then, explain the mistake.

| Question | Which answer? Explain the mistake. |  |
| :---: | :---: | :---: |
| A band held a concert for charity. Adult tickets $=£ 15$ Child tickets $=£ 8$ 250 tickets were sold. 110 of these were child tickets. How much money was raised? | Answer A: $\begin{aligned} & 250 \times £ 15=£ 3750 \\ & 110 \times £ 8=£ 880 \\ & £ 3750+£ 880=£ 4630 \end{aligned}$ | Answer B: $\begin{aligned} & 140 \times £ 15=£ 2100 \\ & 110 \times £ 8=£ 880 \\ & £ 2100+£ 880=£ 2980 \end{aligned}$ |
|  | Explain the Mistake: <br> The mistake in $A$ is they forgot to take the child tikets cumounts off the abults |  |
| 2000 people going to the match. Coaches can fit 60 people. Cars can fit 5 people. There are 25 coaches. | Answer A: $\begin{aligned} & 60 \times 25=1500 \\ & 2000-1500=500 \end{aligned}$ <br> 500 cars needed | Answer B: $\begin{aligned} & 2000-60 \times 25=500 \\ & 500 \div 5=100 \\ & 100 \text { cars needed } \end{aligned}$ |
| How many cars are needed to take everyone to the match? | They wrote the umount of people young in cars, not the amont of cars needed |  |



22 This is the net of a cube.


22 This is the net of a cube.


What is the volume of the cube?

Riley eats 2 biscuits a day. How many does he eat in two years?

Riley eats 2 biscuits a day.

Riley eats 2 biscuits a day. How many does he eat in two

Riley eats 2 biscuits a day. How many does he eat in two years?

Riley eats 2 biscuits a day.

## How many does he eat in two years?

60 minutes $=1$ hour
24 hours $=1$ day
7 days = 1 week

52 weeks $\approx 1$ year
12 months $=1$ year
365 days $=1$ year
(unless a leap year)

The length of an alligator can be estimated by:

- measuring the distance from its eyes to its nose
- then multiplying that distance by 12

What is the difference in the estimated lengths of these two alligators?


Not to scale

The length of an alligator can be estimated by:

- measuring the distance from its eyes to its nose

(a) Add 250 cm
(b) Multiply by 12


Not to scale

The length of an alligator can be estimated by:

- measuring the distance from its eyes to its nose
- then multiplying that distance by 12


Not to scale

The length of an alligator can be estimated by:

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- measuring the distance from its eyes to its nose
- then multiplying that distance by 12

What is the difference in the estimated lengths of these two alligators?


Not to scale

## Method A:

$17.5 \times 12=210 \mathrm{~cm}$
$15 \times 12=180 \mathrm{~cm}$
$210 \mathrm{~cm}-180 \mathrm{~cm}=30 \mathrm{~cm}$

## Method B:

$17.5 \mathrm{~cm}-15 \mathrm{~cm}=2.5 \mathrm{~cm}$
$2.5 \mathrm{~cm} \times 12=30 \mathrm{~cm}$


## Explain the Mistakes

Reflect the shape in the mirror line.

Mistake 1


Mistake 3

mirror
Mistake 2

mirror

Explain the Mistakes


## Which Answer?



## Explain the mistakes.

## Explain the Mistakes

In Roman Numerals, 4 is written IIII

$$
\begin{aligned}
& \hline \mathrm{I}=1 \\
& \mathrm{~V}=5 \\
& \mathrm{X}=10 \\
& \mathrm{~L}=50 \\
& \hline
\end{aligned}
$$

In Roman Numerals, 15 is written VVV

## Which Answer?

## What is the missing Roman Numeral or number?



Explain the mistakes.

| 100 |  | 10 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100 |  | 10 |  | 1 |  |
| 100 |  | 10 |  |  |  |
| 100 |  | 10 | 10 | ) |  |
| 100 | 100 | 10 | 10 | 1 |  |

## 674-256

| 100 |  | 10 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100 |  | 10 |  | (1) |  |
| 100 |  | 10 |  | 1 |  |
| 100 |  | 10 | 10 | ) |  |
| 100 | 100 | 10 | 10 | ) |  |

## 674-256

674

- 256

2

## Correct or Incorrect? <br> $\checkmark$ or $\boldsymbol{x}$

Which of these regroups have been done correctly?


Next Step
In each calculation, what's the remainder? $3 / 5^{2} 8^{1} 2$


## Explain the Mistakes

Example A:


Example B:


Example C:
$\begin{array}{r}80123 \\ -6070 \\ \hline 2053 \\ \hline\end{array}$

Extend: complete the calculations correctly.

## How Many Ways?

Make all the fractions that are more than 0.5 and less than 0.75 using these numbers:

## $\begin{array}{llllll}2 & 3 & 4 & 5 & 6 & 8\end{array}$



Level 1: I can find a way
Level 2: I can find different ways
Level 3: I know how many ways there are

## How Many Ways?

Make a cuboid using 16 to 18 cubes.
There must be at least 4 squares on each face of the cuboid.

Level 1: I can find a way
Level 2: I can find different ways
Level 3: I know how many ways there are


## How Many Ways?



The answer must be a proper fraction.
Level 1: I can find a way
Level 2: I can find different ways
Level 3: I know how many ways
there are







## Explain

## Which angle is larger?



$$
4, \ldots, \quad 22
$$

$$
\begin{aligned}
& 0, \quad 18 \\
& 4, \quad 22 \\
& 4, \quad, \quad 22
\end{aligned}
$$

$$
0, \underline{9}, 18
$$

$$
4, \ldots, 22
$$

$$
4, \ldots, \quad 22
$$

$$
0, \underline{9}, 18
$$

$$
4,13,22
$$

$$
4, \ldots, \quad 22
$$

$$
\begin{aligned}
& 0, \underline{9}, 18 \\
& 4, \underline{13}, 22 \\
& 4, \underline{10}, 16,22
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6= \\
& 100-(10 \times 6)= \\
& (100-10) \times 6=
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=40 \\
& 100-(10 \times 6)=\square \\
& (100-10) \times 6=
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=40 \\
& 100-(10 \times 6)=40 \\
& (100-10) \times 6=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=40 \\
& 100-(10 \times 6)=40 \\
& (100-10) \times 6=540
\end{aligned}
$$



(d) $364+183$
(e) $381+166$
(d) $674-385$
(e) $551-262$
(d) $342 \times 8$
(e) $684 \times 4$
(d) $162 \div 3$
(e) $324 \div 6$
$162 \div 3$
694-365
$162 \div 6$
674-385
$324 \div 6$
551-262

$$
\begin{aligned}
& \frac{1}{10} \text { of } 40=\square \\
& \frac{1}{5} \text { of } 40=\square \\
& \frac{2}{5} \text { of } 40=\square \\
& \frac{4}{10} \text { of } 40=\square
\end{aligned}
$$

$$
\begin{aligned}
& 28 \div 4=\square \\
& 28 \div 4=\square+1 \\
& 28 \div 4=\square \times 1 \\
& 28 \div 4=\square \div 1
\end{aligned}
$$

92. In some schools, pupils were not explicitly taught how to apply the mathematics they had recently learned to mathematical problems. Their only exposure to solving mathematical problems was through answering the final few questions of a predominantly procedure-focused exercise. Often, many pupils did not reach this stage of the exercise. These pupils, therefore, had very little experience of applying mathematical methods beyond routine and established applications. Pupils in these schools were notably less confident when solving mathematical problems.
93. In a minority of schools, problem-solving was explicitly planned into the curriculum. Teachers understood the importance of demonstrating how to apply mathematical methods to problems and giving pupils multiple opportunities to practise applying these methods to structurally similar problems. In the most successful lessons, teachers clearly 'drew out' the similarities between problems to help pupils identify the mathematical techniques that might be useful for different types of problem.

## Contexts

Which questions are answered by calculating $15 \div 5$ ?
(a) There are 15 children in the hall. There are 5 children at each table. How many tables are there in the hall?
(b) 15 people have 5 grapes each. How many grapes in total?
(c) 5 children share 15 pencils. How many pencils each?
(d) Of the 15 children that go to running club, there are 5 girls. How many boys go to running club?

## Interpreting Remainders

(a) Cupcakes are packed in boxes of 6 . The bakery has made 30 cupcakes. How many full boxes can be made?
(b) Cupcakes are packed in boxes of 6 . The bakery has made 32 cupcakes. How many full boxes can be made?
(c) Cupcakes are packed in boxes of 6 . The bakery has made 32 cupcakes. How many boxes are needed to hold all of the cupcakes?

Tom had $\square$
He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $£ 60$ left.

Tom had $\square$
He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $£ 60$ left.

## Money Tom had

Tom had $\square$
He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $\mathbf{£} 60$ left.

## Money Tom had $\quad £ 30$

Tom had $\square$
He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $\mathbf{£ 6 0}$ left.

| Money Tom had |  | $£ 30$ |
| :---: | :---: | :---: |
| Cost of bike | $£ 60$ left |  |

Tom had $\square$
He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $\mathbf{£ 6 0}$ left.

| Money Tom had |  | $£ 30$ |
| :--- | :---: | :---: |
| $£ 60$ bike | $£ 60$ left |  |

## Tom had £90

He was given $\mathbf{£ 3 0}$ for his birthday.
Then he spent half of his money on a bike.
Tom has $£ 60$ left.


My number was
I multiply my number by 4
Then I subtract 15
Now my number is $\mathbf{2 1}$

My number was
I multiply my number by 4
Then I subtract 15
Now my number is $\mathbf{2 1}$


My number was
I multiply my number by 4
Then I subtract $\mathbf{1 5}$
Now my number is $\mathbf{2 1}$


My number was $\square$
I multiply my number by 4
Then I subtract $\mathbf{1 5}$
Now my number is $\mathbf{2 1}$


My number was
I multiply my number by 4
Then I subtract 15
Now my number is $\mathbf{2 1}$


My number was 9
I multiply my number by 4
Then I subtract 15
Now my number is $\mathbf{2 1}$

(a) My number was $\square$ I add 3. Then I multiply by 4. Now my number is $\square$
(b) My number was $\square$ I subtract 3. Then I divide by 2. Now my number is $\square$
(c) My number was $\square$. I add 4. Then I divide by $\mathbf{2}$. Now my number is $1 \mathbf{2}$.
(d) My number was $\square$. I add 4. Then I multiply by 2. Now my number is 12.
(e) My number was $\mathbf{1 5}$. I multiply by 4. Then I subtract $\mathbf{2 5}$. Now my number is $\square$
(f) My number was $\square$. I add 25. Then I divide by 4. Now my number is 15.

'The blue/red number will be larger because...'

## Jen thinks of a number.

She multiplies her number by 3
Then she adds 2
Now Jen's number is $\square$


Jen thinks of a number.

She multiplies her number by 3
Then she adds 2
Now Jen's number is



She adds 2
Then she multiplies her number by 3
Now Jen's number is $\square$


Jen thinks of a number.
She multiplies her
number by 3
Then she adds 2
Now Jen's number is


## Jen thinks of a number.

She multiplies her number by 3
Then she adds 2
Now Jen's number is 20



She adds 2
Then she multiplies her number by 3
Now Jen's number is 24


Jen thinks of a number.

She multiplies her number by 3
Then she adds 2
Now Jen's number is 20


She adds 2
Then she multiplies her number by 3
Now Jen's number is 24

(a) I chose a number. I multiplied my number by 3. Then I added 5. Now, my number is 26 . What number did I choose? 7
(b) I chose a number. I multiplied my number by 3 . Then I added 6 . Now, my number is 27 . What number did I choose? 7
(c) I chose a number. I multiplied my number by 3 . Then I subtracted 6 . Now, my number is 27 . What number did I choose? ||
(d) I chose a number. I divided my number by 3 . Then I subtracted 6. Now, my number is 27 . What number did I choose? 99


The answer to question $D$ is the same/larger/smaller than question $C$ because...

## Task C

My number was $\square$
I multiply/divide my number by 6
| add/subtract 3
Now my number is $\mathbf{2 7}$

What is the largest number that could go in the blue box? 180

What is the smallest number that could go in the blue box? 4

To make the number in the blue box as large as possible...

$$
\text { - and } \div \text {, the startmust be as bigy as possiblef. }
$$

There are $\mathbf{2 4}$ children at running club.
$\frac{3}{4}$ of the children at running club are girls. How many girls go to running club?

There are $\mathbf{2 4}$ children at running club.
$\frac{3}{4}$ of the children at running club are girls. How many girls go to running club?

24 children


There are $\mathbf{2 4}$ children at running club.
$\frac{3}{4}$ of the children at running club are girls. How many girls go to running club?

24 children


There are $\mathbf{2 4}$ children at running club.
$\frac{3}{4}$ of the children at running club are girls. How many girls go to running club?

24 children


Ben had $\mathbf{£ 2 4}$.
Then, he spent $\frac{2}{3}$ of his money on a $t$-shirt.
How much money did Ben have left?

Ben had $£ 24$.
Then, he spent $\frac{2}{3}$ of his money on a $t$-shirt.
How much money did Ben have left?

## £24



Ben had $£ 24$.
Then, he spent $\frac{2}{3}$ of his money on a $t$-shirt.
How much money did Ben have left?

## £24


$\square$ spent $\quad \square$ left

Ben had $£ 24$.
Then, he spent $\frac{2}{3}$ of his money on a $t$-shirt.
How much money did Ben have left?
$£ 24$

£16 spent $£ 8$ left

## Fractions of a Quantity

Zara's book is 60 pages long.
Zara has read $\frac{\mathbf{3}}{\mathbf{4}}$ of her book.
How many pages does Zara have left to read?
Which bar model represents the question?


## Fractions of a Quantity

Which bar model correctly represents each question?


Cut out. Match each question to the appropriate bar model.


Fractions of a Quantity

| Question | Complete the bar model and answer: |
| :---: | :---: |
| (a) Tom has $£ 30$. He spends $\frac{3}{5}$ of his money on a toy. <br> How much does the toy cost? | $£ \mathbf{£ 3 0}$     <br>      |
| (b) There are 30 coins on the table. $\frac{3}{5}$ are showing heads. <br> How many are showing tails? |  |
| (c) Jo has read $\frac{3}{5}$ of her book. <br> She has read 30 pages. <br> How many pages long is Jo's book? |  |

## Fractions of a Quantity

$\begin{array}{lll}\text { (a) } \frac{1}{5} \text { of } 420= & \text { (b) } \frac{1}{6} \text { of } 420= & \text { (c) } \frac{1}{7} \text { of } 420=\end{array}$
(d) 6 plums weigh 420 g . Zack eats one plum.

What is the weight of the remaining plums?
(e) $\frac{5}{6}$ of the 420 children in the school are right-handed.

How many left-handed children are there in the school?
(f) Joy has read $\frac{2}{5}$ of her 420-page book. How many pages does she have left to read?
(g) Adam used $\frac{5}{7}$ of the sugar from the pack in the cake.

There were 420 g sugar left in the pack.
How many grams of sugar are there in the cake?
(h) For every 5 women at the concert, there were 2 men.

There were 420 men at the concert.
How many women were there at the concert?
Extension: Compare questions (g) and (h).

## Compare the Info

The patterns are made with identical rectangles and semi-circles.


Pattern $\mathrm{A}=50$


Pattern B=70

The patterns are made with identical rectangles and semi-circles.


Pattern $\mathrm{A}=50$


Pattern B=70


## Compare the Info

The patterns are made with identical rectangles and semi-circles.


Pattern $\mathrm{A}=50$

One extra rectangle, 20 more $\longrightarrow$ Pattern B = 70


## Compare the Info

The patterns are made with identical rectangles and semi-circles.


Pattern $\mathrm{A}=50$


## Compare the Info

The patterns are made with identical rectangles and semi-circles.


Pattern $\mathrm{A}=50$


The towers are made with identical squares and identical rectangles.


Calculate the height of a rectangle and the height of and a square.


The towers are made with identical squares and identical rectangles.


Calculate the height of a rectangle and the height of and a square.


## Compare the Info

The towers are made with identical squares and identical rectangles.


Calculate the height of a rectangle and the height of and a square.


The towers are made with identical squares and identical rectangles.


Calculate the height of a rectangle and the height of and a square.

$$
\begin{gathered}
\sqrt{\square}=11 \mathrm{~cm} \\
\sqrt{\square}=7 \mathrm{~cm}
\end{gathered}
$$

$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay $\square$ for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay $\square$ for their tickets.

What is the cost of a child ticket to the show? Child ticket $=\square$
$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay $£ 58$ for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show? Child ticket $=\square$
$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay £58 for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show?
Child ticket $=\square$

$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay £58 for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show?
Child ticket $=\square$

$$
A=£ 14
$$


$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay $£ 58$ for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show?
Child ticket $=\square$

$$
A=£ 14
$$


$\mathbf{3}$ adults and $\mathbf{2}$ children go to the show. They pay $£ 58$ for their tickets.
$\mathbf{2}$ adults and $\mathbf{2}$ children go to the show. They pay £44 for their tickets.

What is the cost of a child ticket to the show?
Child ticket $=£ 8$

$$
\begin{aligned}
& A=£ 14 \\
& \text { C }=£ 8
\end{aligned}
$$



Compare the Info
Task B Version 1

| Question | Complete the bar model and answer: |
| :---: | :---: |
| 3 apples and $\mathbf{2}$ pears cost $£ 1.40$ <br> 2 apples and 2 pears cost $£ 1$. 10 <br> What is the cost of a pear? | Clue: Start by calculating the cost of an apple $\begin{aligned} & \text { apple }=30 p \\ & \text { pears }=50 p \rightarrow 7 \div 1 \text { pear }=25 p \end{aligned}$ |
|  | Clue: Pattern B has $\qquad$ 2 more triangles than Pattern A $\begin{aligned} & 30-2 h=8 \\ & \times 2 \text { triungles }=2 h \rightarrow \div 2=11 \end{aligned}$ |
| The rectangles are identical. <br> Length $=$ $\square$ <br> Width $=$ | Choose: Start by calculating the ength $O$ width $\begin{aligned} & \frac{18 \div 2=9}{83-18=15} \\ & \frac{1515,51}{15 \mathrm{~cm}} \times 5=15 \end{aligned}$ |

Compare the Info


There are 28 pupils in a class.
The teacher has 8 litres of orange juice.
She pours 225 millilitres of orange juice for every pupil.


How much orange juice is left over?

